Extensions of Standard Hough Transform based on Object Dual and Application

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Abstract—This paper proposes a set of Standard Hough Transforms for the recognition of naif, standard, thin analytic straight line. The Standard Hough Transform is applied to some objects to obtain the data of the accumulator in order to permit the detection. A Hough Transform based on the thickness of analytic straight line is defined and an experiment result of the standard Hough Transform based on a square is also presented in this paper. The proposed methods take the recognition in noisy images into account. The proposed methods could be implemented and added to the functions of image processing software.

Keywords: Hough Transform, Discrete geometry, Pattern Recognition, Reconstruction

1. Introduction

Hough Transform is a classic method used to detect line in a noisy picture. The method has been introduced by Paul Hough in 1962. During long time, the method has been used and adapted by scientists to the recognition of digital circles, ellipse and others shapes\cite{1,2,8}. The method uses an image space and a parameter space.

Works on errors occurring in the application of the Hough Transform have also been proposed by \cite{6}. The digitalization of the image space and the parameter space create some errors in the precision of objects recognition.

Another variant of Hough Transform is the standard Hough Transform that associates a point \((x, y)\) in an image space to a sinusoid curve \(p = x\cos \theta + y\sin \theta\) in the parameter space. The Standard Hough Transform consider that a point belonging to two sinusoid curves in a parameter space corresponds to a line in an image space crossing two points, with each point associated to each curve.

Others works on Hough transform have been proposed by Henri Matre\cite{8} to unify the Hough Transform definition and adapted by scientists to the recognition of digital circles, ellipse and others shapes\cite{1,2,8}. The method uses an image space and a parameter space.

The recognition of analytic straight line in the parameter space is studied. We focus on the

2. Analytic straight line

There are several definitions of analytic hyperplanes\cite{1,5,7,13}.

Definition 1. (Analytic hyperplane \cite{1,5,7,13}) Let \(H\) be a analytic hyperplane in dimension \(n\) noticed \(\mu \sum_{i=1}^{n} (A_i x_i) + \omega \) with the parameters \(A(A_1, A_2, \ldots, A_n, A_n) \in \mathbb{R}^n, \mu \in \mathbb{R} \) and \(\omega \in \mathbb{R}\), then:

- \(H\) is called naif if \(\omega = \max_{1 \leq i \leq n} (|A_i|)\)
- \(H\) is called standard if \(\omega = \sum_{i=1}^{n} (|A_i|)\)
- \(H\) is called thin if \(\omega \max_{1 \leq i \leq n} (|A_i|)\)
- \(H\) is called thick if \(\omega \sum_{i=1}^{n} (|A_i|)\)
- \(H\) is called *-connected if \(\max_{1 \leq i \leq n} (|A_i|) j \omega \sum_{i=1}^{n} (|A_i|)\)

If \(\omega \max_{1 \leq i \leq n} (|A_i|), \) thin hyperplane is decomposed : some digital points are not k-neighbors. In dimension 2, we obtain the definition of decomposed line. The standard line is a particular case in 2D of a generalized standard hyperplane in dimension \(n\). The naive line is also the same for the generalized naive hyperplane in dimension \(n\).

According to the definition of the standard straight line \(\frac{|a| + |b|}{2} \leq ax + by \leq \frac{|a| + |b|}{2}\), the thickness is determined by \(\omega = a \rightarrow b\). That lead to set a contraint to the recognition in the parameter space. A digital point of a standard line has its Hough transform passing through the same cell of the accumulator. This contraint could be used to build a quantization of the parameter space. It is the same for the naive straight line \(\frac{\max(a,b)}{2} \leq ax + by \leq \frac{\max(a,b)}{2}\) where thickness is determined by \(\omega = \max(a,b)\).

Proposition 2. For the standard Straight line \(\frac{|a| + |b|}{2} \leq ax + by \leq \frac{|a| + |b|}{2}\), we have:

\(\frac{|a| + |b|}{2\sqrt{a^2 + b^2}} \leq \rho \leq \frac{|a| + |b|}{2\sqrt{a^2 + b^2}}\) and \(\delta \rho = \frac{\omega}{\sqrt{a^2 + b^2}}\) with \(\omega = a \rightarrow b\).
have also δR space (a sub-set of Definition 4. by [1], [12]). The concepts of this method [1] are shortly defined by:

Several methods are proposed in the sections 3, 4 on the points of an object with the Standard Hough Transform?

Proposition 3. For the naive straight line

\[ -\frac{\max(a,b)}{2} \leq ax+by \leq \frac{\max(a,b)}{2} \]

we have:

\[ p \leq \frac{\max(a,b)}{2\sqrt{a^2+b^2}} \quad \text{and} \quad \delta p = \frac{\omega}{\sqrt{a^2+b^2}} \]

Proof: By analogy to the standard straight line proof, we have also \( \delta p = \frac{\omega}{\sqrt{a^2+b^2}} \) with \( \omega = \max(a,b) \).

We conclude that in any case of a standard straight line or a naive straight line, each center of a pixel has its p value in the interval \([-\frac{a}{\sqrt{a^2+b^2}}, \frac{a}{\sqrt{a^2+b^2}}\)] with \( \frac{\omega}{\sqrt{a^2+b^2}} \) and \( \sin \theta = \frac{b}{\sqrt{a^2+b^2}} \).

The interval \([-\frac{a}{\sqrt{a^2+b^2}}, \frac{a}{\sqrt{a^2+b^2}}\)] and \( \theta \) with \( \cos \theta = \frac{a}{\sqrt{a^2+b^2}} \) and \( \sin \theta = \frac{b}{\sqrt{a^2+b^2}} \) must be well represented in the parameter space to reduce errors in the detection in the image space.

Moreover, when \( \omega = -a+b \) in the case of standard straight line, we will have \( \frac{1}{2} \leq \frac{\omega}{\sqrt{a^2+b^2}} \leq 1 \) that is \( 1 \leq \frac{\omega}{\sqrt{a^2+b^2}} \leq 2 \). It is clear \( \frac{\omega}{\sqrt{a^2+b^2}} \geq 1 \) because of \( a^2 + b^2 + 2|a||b| > a^2 + b^2 + 2|a||b| \geq 0 \). Suppose that \( \frac{\omega}{\sqrt{a^2+b^2}} > 2 \). We have \( \omega^2 > 4(a^2 + b^2) \). As \( \omega = |a| + |b| \), one obtains \( a^2 + b^2 + 2|a||b| > 4(a^2 + b^2) \). So \( 2|a||b| \) \( > 3(a^2 + b^2) \) and \( |a||b| \) \( > \frac{3}{2}(a^2 + b^2) \). That is absurd. We conclude that \( 1 \leq \frac{\omega}{\sqrt{a^2+b^2}} \leq 2 \) and \( \frac{1}{2} \leq \frac{\omega}{\sqrt{a^2+b^2}} \) \( \leq 1 \).

If \( \omega = \max(a,b) \) in the naive straight line case, one will have \( 0 \leq \frac{\omega}{\sqrt{a^2+b^2}} \leq \frac{1}{2} \) and \( 0 \leq \frac{\omega}{\sqrt{a^2+b^2}} \leq 1 \).

Thus, we can see the influence of the standard or naive straight line on the parameter space.

How do the recognition of analytic straight line does based on the points of an object with the Standard Hough Transform? Several methods are proposed in the sections 3.4

3. Extensions of Standard Hough Transform

An extended Standard Hough Transform has been presented by [11], [12]. The concepts of this method [11] are shortly explained in this section.

Definition 4. (Dual of a point [11], [12]) Let I and P be respectively an image space (a sub-set of R²) and a parameter space (a sub-set of R²). Let M(x, y) be a point in I, the hough transform of the point M is set of point (θ, p) belonging to P verifying \( p = x \cos(\theta) + y \sin(\theta) \).

The question is how to use the previous definition of the dual of a continuous point in the discrete space case. That permit to work on the dual of a pixel.

Before getting the dual of a pixel, the dual of a segment is defined by:

Theorem 5. (Dual d’un segment) The dual of a segment is the area limited by the dual of its vertices.

The prove of the theorem [5] is established by SERE and others in [11].

A pixel is a square containing a set of continuous point. The idea is to determine the union of the dual of continuous point.

The theorem 6 determines easily the dual of each point in the area of a pixel.

Theorem 6. (Dual of a pixel [11]) The dual of a pixel is the union of the dual of its diagonals segments.

The proove of the theorem 6 is established by SERE and others in [11].

Definition 7. (Preimage [11]) Let \( S = \{ p_1, p_2, ..., p_{n-1}, p_n \} \) be a set of n pixels \( P_i \) in \( \xi_2 \). The preimage of S is defined by \( \text{Preimage}(S) = \bigcap_{1 \leq i \leq n} \text{Dual}(P_i) \).

The proposed methods are based on the dual of objects such as rectangles, triangles, squares. The recognition of Standard straight line is based on the dual of a square. The naive straight line is related to the dual of a losange, a part of a square. Many object could been used to realize the detection of thin analytic straight line.

Moreover, thin analytic straight line recognition is a good model for a noisy straight line. The recognition of a noisy straight line in a picture could be seen under several cases. First, During long year, Hough transform definition has given an alternative with an accumulator to get parameters of straight line in a noisy picture. The missing pixels of an image have an influence on the accumulator data. The peak cells will occurs more clear if we have less missing pixels building a straight line, due to the inclusion of preimage [12]. When the number of missing pixel increases, according to the definition preimage [11], [2], [7], parameters cells of straight line verifying the preimage exist in the accumulator. These parameters define some lines that pass through some missing pixels which are not necessary. The thin analytic straight line definition gives a solution to avoid this problem limiting the number of missing pixels in the overcome of the parameters of straight line. These news methods based on the Hough Transform of object included in square permit the detection of thin analytic straight line.

A. Standard Hough Transform based on square

The square Hough Transform is the extended Standard Hough Transform that permit the recognition of standard or naive straight line [11]. Several cases of a square are possible (see the figure 3.1) : a square could been a losange in a pixel or a subset of a square.
When the considered object for the dual is a rectangle, the method is called the rectangular Hough Transform.

B. Rectangular Hough Transform

The dual of a rectangle has been proposed by SERE and al. in [1], [12]. It is the union of the dual of each point belonging to the rectangle. This set is easily determined by the theorem 8.

**Theorem 8. (Dual of a rectangle)** the dual of a rectangle is the union of the dual of the diagonals segments.

The proof of the theorem 8 is established by SERE and al. in [1]. The rectangular Hough Transform uses the dual of a rectangle to build the data of the accumulator. A rectangle can be a subset of a square.

**Theorem 9.** The rectangular Hough Transform will consist of thin analytic straight line recognition, if the rectangle is a part of the losange related to the naive analytic straight line recognition.

The figure 3.2 shows an illustration of the dual of a rectangle: we see in 3.2a a rectangle with continuous points, in 3.2b the dual of elementary points, and in 3.2c the dual of a rectangle.

A grid can be composed of vertical rectangles or horizontal rectangles and a rectangle can also be a subset of a square.

An example of a preimage of rectangles is showed by the figure 3.3.

C. Triangular Hough Transform

The triangular Hough Transform is based on the dual of a triangle in the accumulator. If a triangle is a part of a pixel, the triangular Hough transform will give results of thin analytic line recognition. The dual of a triangle has been established by SERE and al. in [1], [12]. The theorem 10 shows how to obtain the dual of a triangle.

**Theorem 10. (Dual of a triangle)** The dual of a triangle is the union of the dual of its adjacent sides.

The figure 3.4 shows in 3.4a a triangle and the dual of its vertex in 3.4b. We have a dual of a triangle in 3.4c showing the application of the theorem 10.

The proposed methods detect thin analytic straight line because the object used for the dual is a subset of a pixel or a square. That leads to have the thickness of analytic straight line less important than the thickness of the Standard one.

A combination of these methods will be also possible, if the grid is composed of triangles, rectangles or squares. Others possibilities are to extend the dual of a pixel proposed by Martine Dexet [2].

4. Generalized dual space

An extension of the dual of Martine Dexet in [2] permits thin analytic straight line recognition in doing an application.
A. Dual of a concave object

This section deals with the dual of a concave object. Our idea concerns the concave object that is crossed by a hyperplane in 2D space by a straight continuous line.

In [12], we have established a model to determine the dual of a concave object. If the grid is irregular with concave objects, the dual of a concave object will be determined by the dual of its convex hull.

The convex hull is defined by:

**Definition 11. (Convex Hull)** Let \( O \subseteq \mathbb{R}^n \) be a digital object, the convex hull of \( O \) noticed \( \text{conv}(O) \) is the smallest convex part of \( \mathbb{R}^n \) that contains \( O \). We have: \( \forall (A, B) \subseteq \text{conv}(O) \) then \( [A, B] \subseteq \text{conv}(O) \) where \( \text{Conv}(O) \) correspond to the convex part of \( O \).

Then, we propose a definition of a dual of a concave object:

**Proposition 12. (Dual of a concave object)** Let \( O \subseteq \mathbb{R}^n \) a digital object. There is not a continuous hyperplane \( H \) with \( H \cap O = \emptyset \) splitting \( O \) into two digital objects. The dual of \( O \) is equal to the dual of \( \text{conv}(O) \).

The proof of this proposition is in the thesis of SERE [12]. The proposition [12] permit to convert a concave object into its convex hull to obtain its dual.

The figure 4.1 shows an illustration of the proposition 12.

**Figure 4.1:** Dual of a concave object and its convex hull

5. Experiment result of the Standard Hough Transform based on a square

This section concerns experimental results of the extension of the Hough Transform based on a square. The parameter space is modelized by an accumulator, a 2D matrix that has its own size depending on the initial size of the image space. Let \( I \) be the image with a \( H \times L \) size. The parameter space size is determined by \( \sqrt{H^2 + L^2} \).

The method determines the dual of the diagonal of a square based on the theorem 6. The center of cells of the accumulator between the limited curve of the dual is selected and its value is increased by one.

The figure 5.1 is the dual of a pixel of center \((5, 10)\).

**Figure 5.1:** The standard Hough Transform of a pixel \((5, 10)\)

The figure 5.2 presents some parameter points in the parameter space and their corresponding analytic straight line in color in the image space. It shows the dual of each pixel of center \((i, j)\) of the analytic straight line \(-1 \leq i-j \leq 1\).

**Figure 5.2:** The standard Hough Transform of each pixel of an analytic straight line \(-1 \leq i-j \leq 1\) and analytic straight lines corresponding to some points of the parameter space.

The figure 5.3 shows illustrations of the detection of an analytic straight line. In these both images, there are on the right image, the initial line in white color and on the left image, the result of the detection with several colored lines found crossing on the initial white line.
6. Conclusion

This paper shows extensions of standard Hough Transform to detect analytic straight line with experiment result of the standard Hough transform applied to a square. The proposed methods use a quantization of the parameter space taking the recognition in noisy image into account.

In perspectives, the experiment result of others proposed methods and comparisons between them still remain to be done.

References


