

Sensitivity Analysis of Process Parameters on the Optimal Mix and Profit Margin in Custard Production

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ABSTRACT

In this study, time studies of the processes involved in custard production as well as the costs implications were analysed. The results obtained were transformed into a format which enabled simplex optimization technique to be performed on the system. Thereafter, the system was programmed using MATLAB Graphic Users Integrated Development Environment (GUIDE) for simplex optimization to take care of possible changes in future in both the objective function and the constraints and to also generate quicker results. Holistic sensitivity analysis was carried out using each of the sensitive parameters and the results obtained in each case were analysed. The insight from the study will significantly enhance the decision making process of custard production industries. The sensitivity analysis results from this study can serve as a pedagogical tool of system changes to parameter variations.

Keywords: *Sensitivity Analysis, Custard Production, Simplex Optimization, Simulation*

1. INTRODUCTION

For more than two decades now, manufacturing industries worldwide have seen a revolution in the way they operate. Consumers have become more and more demanding, and the key to firm survival is the recognition of the importance of customer satisfaction. Consequently, companies have been forced to enhance the quality of both their processes and products (Efstratiadis et al., 2000). The focus of this study, the food industry, has also become increasingly multifaceted and competitive in recent years (Chong et al., 2001; Knowles et al., 2004; Spiegel et al., 2006; USOCDD, 2007). In this environment, company managers have to deal with a number of problems. Sales are slowing down and operating costs are increasing, while customers are becoming more demanding and selective (Henchion & McIntyre, 2005). Therefore, industrial managers must thus consider how to maintain profitability in a shrinking market, while providing increasingly sophisticated customers with high quality products and efficient service. In attempting to achieve this seemingly impossible objective, firms can pursue two strategic avenues. First, they can focus on ways to improve the operational efficiency of the system. Second, they can take actions to enhance its operational quality.

In the light of the above, the primary motivation that triggered off this study was when an optimal production mix was sort, which will increase the profit margin at the lowest input cost for custard production industries. It is a known fact that industries at all levels are faced with the challenge of producing the right quality and quantity of their products at the right time and more especially at minimum cost and maximum profit for their survival and growth. Thus, this demands an increase in productive efficiency of the industry. According to Mustapha (1998), the primary responsibility in a production outfit is to make decisions that will answer the question of what size of product should be produced, what production mix will yield the highest profit? What technology or material should be invested upon? What price should be negotiated for the items, etc. In support of

the above, Carson and Maria (1997) observed that Simulation optimization entails finding optimal settings of the input variables which optimize the output variable (s).

Having noted that maximizing profits is generally one of the most important objectives of any production operation, it is important to note that this involves not only making decisions that potentially impact revenues and costs of the firm, but also involves the implementation of solutions in a manner that is both efficient and effective. In tackling this matter therefore, it is crucial to determine the optimum production mix. But the immediate pertinent question that will arise is what determines whether or not a product mix is optimum or not? In answer to this question, Bender, (2000) stated that the question is complicated by the fact that an optimal production mix for one product may not be the optimal production mix for another. It encompasses many things including: utilizing resources in their most efficient and productive manner; providing favorable cash flows; satisfying attractiveness constraints of buyers and or production; maximizing profits in the short and long run and satisfying current demand trends and preventing oversupply situation. The real challenge is to find a production mix that accomplishes all or most of the above.

2. MATERIALS AND METHODS

This study discovered that LCI and KGFI which served as the case studies both experience most of the problems stated above. The industries are faced with the problem of what size of custard to produce in order to satisfy the customers and at the same time make maximum profit from the production processes rather than produce the different sizes at random whenever it is depleted in the warehouse as well as the cost implications.

Simulation of an optimized method was used in this study. This method of simultaneous simulating and optimizing has been applied to various applications, including applications with a single objective,

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applications that require the optimization of multiple criteria and applications with non-parametric objective functions. Azadivar and Lee (1988) applied a simulation optimization algorithm based on Box's complex search method to optimize the locations and inventory levels of semi-finished products in a pull-type production system. Hall et al. (1996) used Evolutionary Strategies (ES) with a simulation model for optimizing a Kanban sizing problem. Lutz (1995) developed a procedure that combined simulation with Tabu Search (TS) to deal with problems of Work-in-Process (WIP) inventory management. Fu and Healy (1992) applied the Perturbation Analysis (PA) technique to inventory models where the demand has an associated renewal arrival process. Okolie et al. (2010) have applied it in determining the production mix for bread industries. Okonkwo and Obaseki (2011) applied it in inventory policy differentiated by demand lead time through rationing. In the study, simplex optimization tool was used because it is a mixed production system and the solution technique is an effective and efficient tool for optimization. The industrial scenario was simulated using a MATLAB and Graphic Users Integrated Development Environment (GUIDE), which is capable of detecting, solving and controlling the problems of optimization in the industries.

The focus of this study was to carry out sensitivity analysis. Sensitivity analysis looks at the effect of the specific data on the results (Chinneck, 2000). Sensitivity analysis asks how sensitive the final results are to variations in data such as the profit per item produced. It may show, for example, that the final solution changes very little even with a large variation in profit per item produced (Okonkwo, 2009). In this case one heaves a sigh of relief. On the other hand if it turns out that the results change dramatically when the estimated profit per item produced changes very slightly, then one have cause for worry, especially if there are millions of naira depending on the outcome of the analysis! In this case there is need to run a number of scenarios showing how things will turn out at various values of profit per item produced in order to arrive at a final recommendation.

3. PRODUCTION PROCESSES IN CUSTARD PRODUCTION

Manufacturing processes of custard producing industries are standardized and simple. They have the following labour time: premixing process, mixing, weighing, sealing, packaging and bagging.

The flowchart for custard production processes are shown in Fig 1.

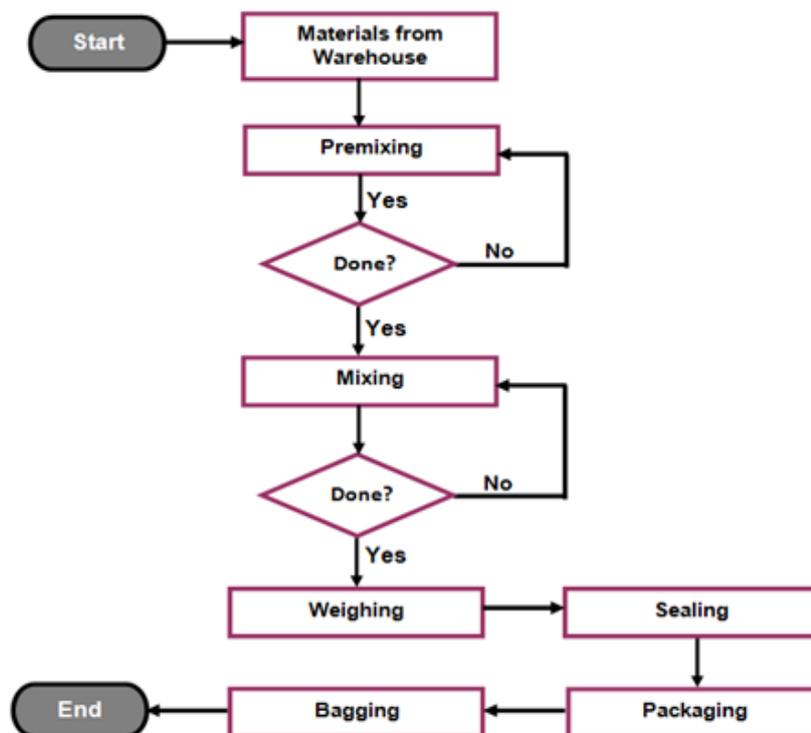


Fig. 1: Flowchart for Custard Production Processes

3.1 Steps for Simplex Optimization

To apply the simplex/iterative method, it is necessary to state the problems in the form in which the inequalities in the constraints have been converted to

equalities because it is not possible to perform arithmetic calculations upon an inequality. The inequalities in

maximization problems are converted to equalities with the aid of slack variables to the left hand side of each



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inequality. The slack variables in maximizing problems represent any unused capacity in the constraint and its value can take from zero to the maximum of that constraint. Each constraint has its own separate slack variable. The inequalities in the minimization problems are converted into equalities by subtracting one surplus variable.

The simplex method for linear programming model follows the under listed steps:

- i) Design the sample problem.
- ii) Setup the inequalities describing the problem
- iii) Convert the inequalities to equations adding slack variables.
- iv) Enter the inequalities in a table for initial basic feasible solutions with all slack variables as basic variables. The table is called simplex table.
- v) Compute C_j and P_j values for this solution where C_j is objective function coefficient for variable j and P_j represents the decrease in the value of the objective function that will result if one unit of variable corresponding to the column is brought into the solution.
- vi) Determine the entering variable (key column) by choosing the one with the highest $C_j - P_j$ value.
- vii) Determine the key row (outgoing variable) by dividing the solution quantity values by their corresponding solution quantity values by their corresponding key column values and choosing the smallest positive quotient. This means that we compute the ratios for rows where elements in the key column are greater than zero.
- viii) Identify the pivot element and compute the values of the key row by dividing all the numbers in the key row by the pivot element. Then change the product mix to the heading of the key column.
- ix) Compute the values of the other non-key rows
- x) Compute the P_j and $C_j - P_j$ values for this solution.
- xi) If the column value in the $C_j - P_j$ row is positive, return to step (vi).
If there is no positive $C_j - P_j$, then the final solution has been reached

3.2 Mathematical Presentation of the Linear Programming Model

$$\text{Max. } P = \sum_{j=1}^n C_j X_j$$

Subject to the linear constraints
 $\sum_{j=1}^n a_{ij} X_j \leq T_i; i = 1, 2, \dots, m$

And $X_j \geq 0; j = 1, 2, \dots, n$

With n decision variables and m constraints the equation can be generalized in the following form:

$$\text{Max. } P = ax_1 + bx_2 + cx_3$$

$$dx_1 + ex_2 + fx_3 \leq T_1$$

$$gx_1 + hx_2 + ix_3 \leq T_2$$

$$jx_1 + kx_2 + lx_3 \leq T_3$$

$$mx_1 + nx_2 + px_3 \leq T_4$$

$$qx_1 + rx_2 + sx_3 \leq T_5$$

$$ux_1 + vx_2 + wx_3 \leq T_6$$

$$(x_1, x_2, x_3 \geq 0)$$

where x_1, x_2, x_3 are the non-basic variables; and $T_1, T_2, T_3, T_4, T_5, T_6$ are the total available time which will be determined after the time study. To solve the mathematical set up model shown above using the simplex method, it requires that the problem be converted into its standard form. Linear program problem should have the following characteristics

- i. All constraints should be expressed as equations by adding slack variables or surplus variables.
- ii. The right-hand side of each constraint should be made non-negative if it is not already this should be done by multiplying both sides of the resulting constraints by -1.
- iii. The objective function should be of the maximization type.

Equation (1) can be expressed as;

$$\text{Max } P = \sum_{j=1}^n C_j X_j + \sum_{i=1}^m OS_i$$

Subject to the constraints

$$\sum_{j=1}^n a_{ij} X_j + S_i = T_i; i = 1, 2, \dots, m$$

And $X_j, S_i \geq 0$, for all i and j

Thus; $\text{Max. } P = ax_1 + bx_2 + cx_3 + OS_1 + OS_2 + OS_3 + OS_4 + OS_5 + OS_6$

Subject to the constraints;

$$dx_1 + ex_2 + fx_3 + S_1 + OS_2 + OS_3 + OS_4 + OS_5 + OS_6 = T_1$$

$$gx_1 + hx_2 + ix_3 + OS_1 + S_2 + OS_3 + OS_4 + OS_5 + OS_6 = T_2$$

$$jx_1 + kx_2 + lx_3 + OS_1 + OS_2 + S_3 + OS_4 + OS_5 + OS_6 = T_3$$

$$mx_1 + nx_2 + px_3 + OS_1 + OS_2 + OS_3 + S_4 + OS_5 + OS_6 = T_4$$

$$qx_1 + rx_2 + sx_3 + OS_1 + OS_2 + OS_3 + S_4 + OS_5 + OS_6 = T_5$$

$$ux_1 + vx_2 + wx_3 + OS_1 + OS_2 + OS_3 + S_4 + OS_5 + OS_6 = T_6$$

$$x_1, x_2, x_3, S_1, S_2, S_3, S_4, S_5, S_6 \geq 0 \text{ [non-negative]}$$

$$x_1, x_2, x_3, S_1, S_2, S_3, S_4, S_5, S_6 \geq 0 \text{ [non-negative]}$$

$$x_1, x_2, x_3, S_1, S_2, S_3, S_4, S_5, S_6 \geq 0 \text{ [non-negative]}$$

$$x_1, x_2, x_3, S_1, S_2, S_3, S_4, S_5, S_6 \geq 0 \text{ [non-negative]}$$

$$x_1, x_2, x_3, S_1, S_2, S_3, S_4, S_5, S_6 \geq 0 \text{ [non-negative]}$$

$$x_1, x_2, x_3, S_1, S_2, S_3, S_4, S_5, S_6 \geq 0 \text{ [non-negative]}$$

where x_1, x_2 and x_3 are quantities of the big custards, medium custards and small custards respectively called the non-basic variables.

S_1, S_2, S_3 and S_4 are the slack variables used to eliminate the inequalities generated in the objective function of the LP model set up.

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P_j = Expected profit to be made after optimization called the total gross amount for outgoing profit.

$C_j - P_j$ = Net evaluation row for the objective function of the LP model called decision variable.

C_j = objective function coefficients

$T_1, T_2, T_3, T_4, T_5, T_6$ = Total available time constants

a, b, c, d, e, f, g, h, i, j, k, l, m, n, p, q, r, s, u, v and w are the process available time constants.

3.3 Graphical Users Interface of the Simulator

The above mathematical model was programmed using MATLAB Graphic Users Integrated Development Environment (GUIDE) for simplex optimization. This is to take care of possible changes in future in either of the constraints and also generate quicker results. The graphical user interface of the developed simulator is shown in Fig 2. It is designed to

be operated through a graphical user interface environment. This means that the user selects a command from the computer screen display by moving a pointer and clicking a menu, sub menu or an icon. With this in place, researchers, planners/managers and modelers that are non programmers can leverage advanced constraint-based solving technology, to ensure that production mix optimization is feasible. They can be confident of the ability to execute their plans and meet demand commitments to customers, through quick and easy execution of programmes, without any consultation to programmers.

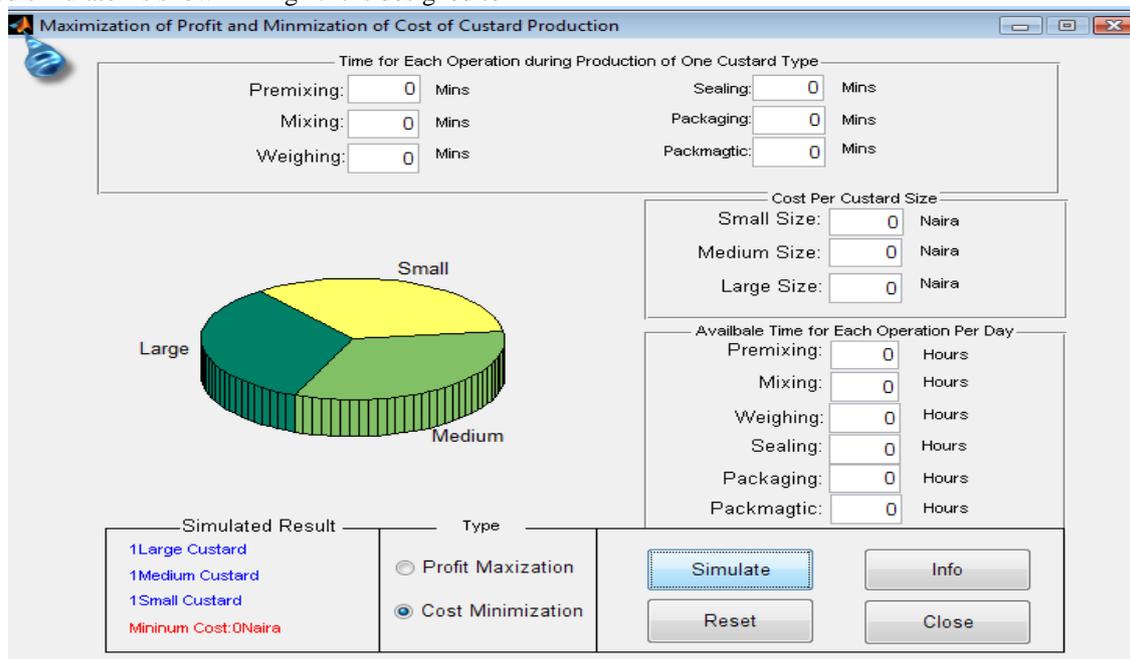


Fig 2: Simulator Graphical User Interface

Whenever a simulation is run, MATLAB Guide Window produces several output files that contain simulation results in pie chart format representing the percentage of large custard (x_1), medium custard (x_2), and small custard (x_3) sizes and quantity to be produced at the end of each production day and expected net profit. It equally has the ability of detecting non-feasible production mix.

3.4 Sensitivity Analysis

Sensitivity analysis of a model can help determine relative effects of model parameters on model results. In other words, the purpose of sensitivity testing of a model is to investigate whether a slight perturbation of the parameter values will result in a significant

perturbation of the model results, that is, the internal dynamics of the model. In this study therefore, sensitivity analysis was carried for all the six sensitive parameters and the results generated were analysed.

3.5 Premixing Time Sensitivity Analysis Results

Simulation runs were carried out with the time of premixing varied from 16 to 30, while other input parameters were kept constant. The dataset for premixing time sensitivity analysis that were kept constant during the simulation run are: [mixing = 20mins, weighing = 3mins, sealing = 3mins, packaging = 3mins, bagging = 3mins, weight of large custard = 1.6kg, weight of medium custard = 1.2kg, weight of small custard = 0.33kg].

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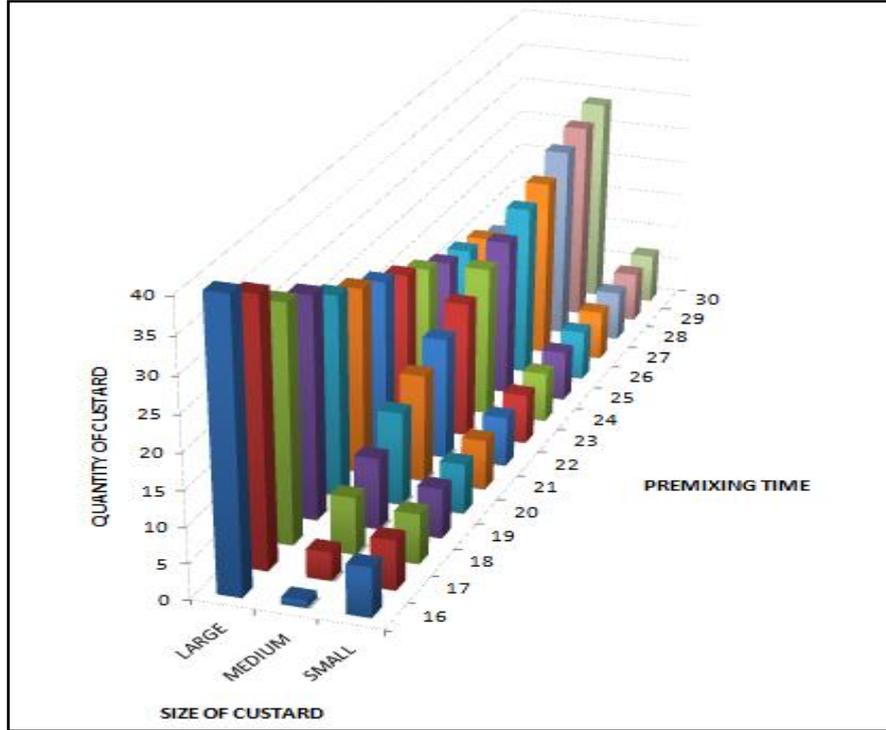


Fig 3: Variation of Premixing Time and Effects on Quantity of Various Sizes of Custard

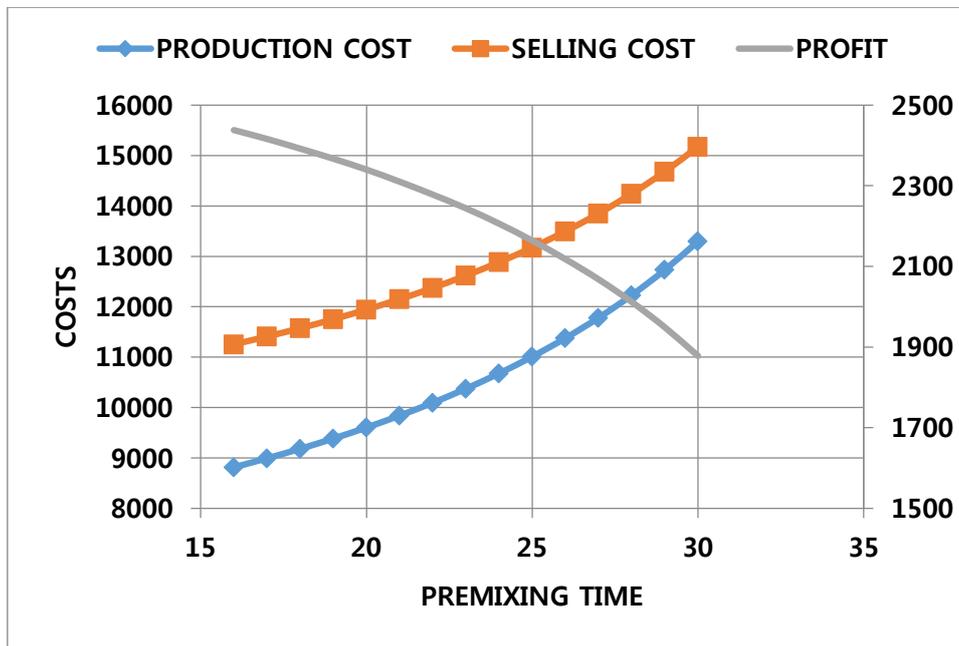


Fig 4: Production Cost, Selling Cost and Profit on Premixing Sensitivity Analysis

It is observed that the time for premixing of the materials for production was varied and the results of the optimal number of custards for the various sizes as well as the cost of production, selling cost and the profit obtained from the simulator were noted. Fig. 3 shows the 3-D column chart in which at each instance of variation of the premixing time, the quantity of custard for each size was shown. From Fig. 4, it is clearly shown that the production cost and selling cost plotted with the primary

axis increase as the premixing time increases. However, the profit made which is the difference between the production cost and selling cost at each instance decreases as the premixing time increases. The time for premixing yielded the highest profit at the lowest premixing time and the lowest profit at the highest premixing time as shown from the graph on the secondary axis of Fig 4. It can therefore be deduced that if the time

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for premixing is reduced then there will be more time for other processes hence the increase in output.

3.6 Mixing Time Sensitivity Analysis Results

Simulation runs were carried out with the time of mixing varied from 13 to 24, while other input parameters were kept constant. The dataset for mixing time sensitivity analysis that were kept constant during the simulation run are: [premixing = 20mins, weighing = 3mins, sealing = 3mins, packaging = 3mins, bagging = 3mins, weight of large custard = 1.6kg, weight of medium custard = 1.2kg, weight of small custard = 0.33kg].

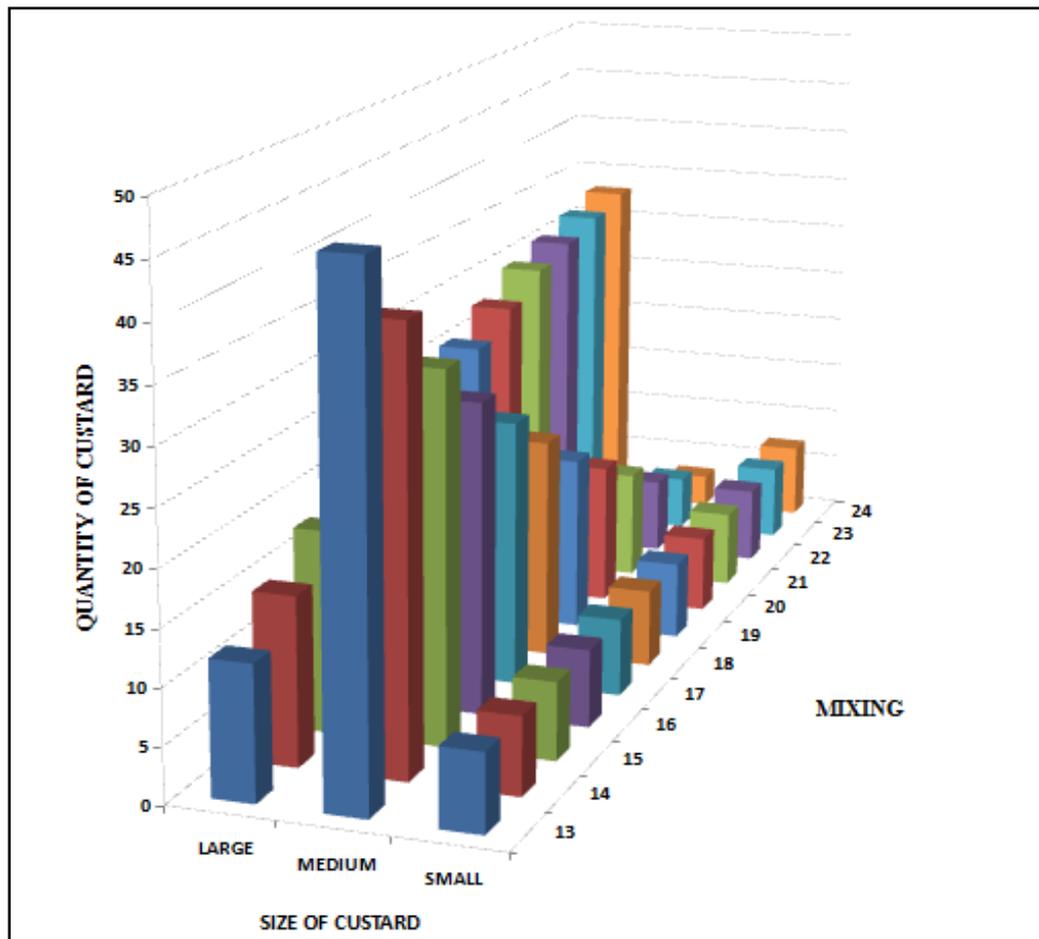


Fig 5: Variation of Mixing Time and Effects on Quantity of Various Sizes of Custard

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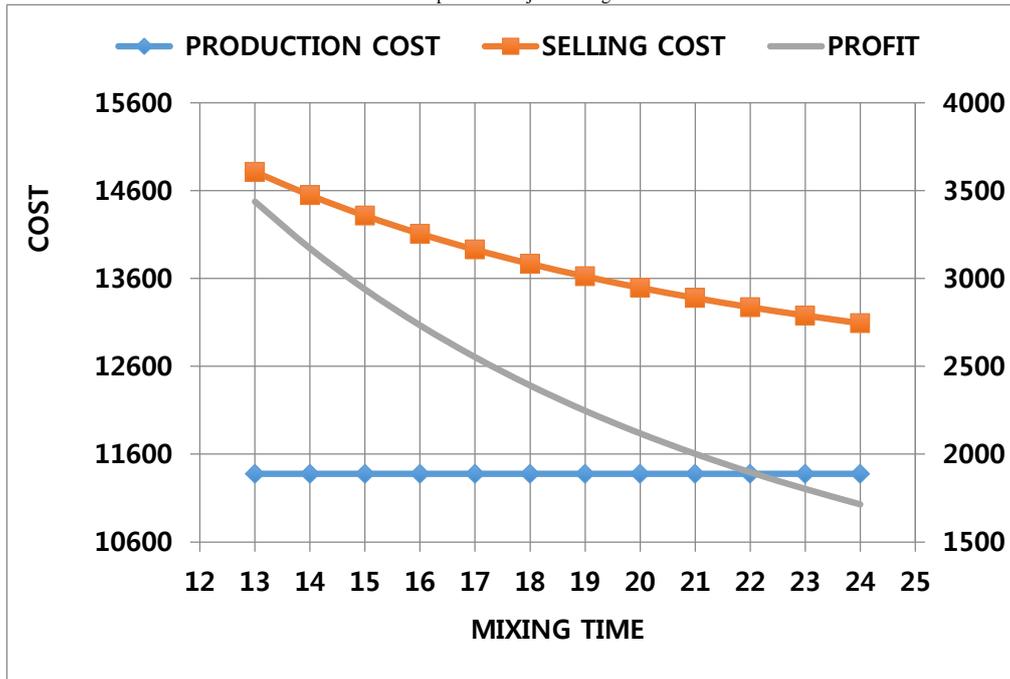


Fig 6: Production Cost, Selling Cost and Profit on Mixing Sensitivity Analysis

Fig. 5 shows the 3-D column chart in which at each instance of variation of the mixing time, the optimal quantity of custard for each size was determined. It is observed that the time for mixing of the materials for production was varied and the results of the optimal number of custards for the various sizes as well as the cost of production, selling cost and the profit obtained from the simulator were noted as shown in Fig. 6, it is seen that the highest quantity was produced when the time used for mixing was least and also at that point, the highest profit was made. It is also observed that the minimum cost of production did not change since the same quantity can be used to achieve optimum production

without altering the quantity of materials that the machine can accommodate for a batch production.

3.7 Weighing Time Sensitivity Analysis Results

Simulation runs were carried out with the time of weighing varied from 1 to 15, while other input parameters were kept constant. The dataset for mixing time sensitivity analysis that were kept constant during the simulation run are: [premixing = 20mins, mixing = 20mins, sealing = 3mins, packaging = 3mins, bagging = 3mins, weight of large custard = 1.6kg, weight of medium custard = 1.2kg, weight of small custard = 0.33kg].

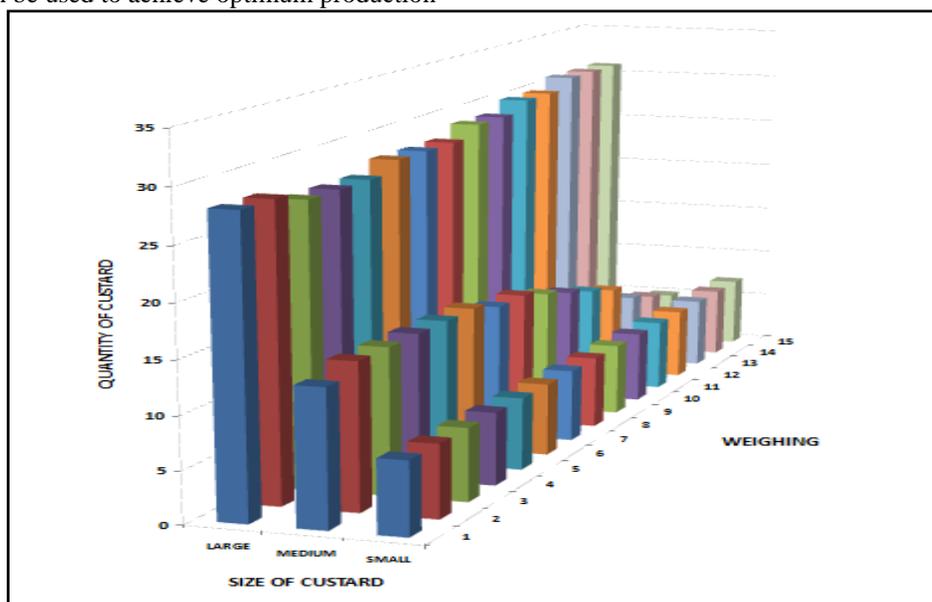


Fig 7: Variation of Weighing Time and Effects on Quantity of Various Sizes of Custard

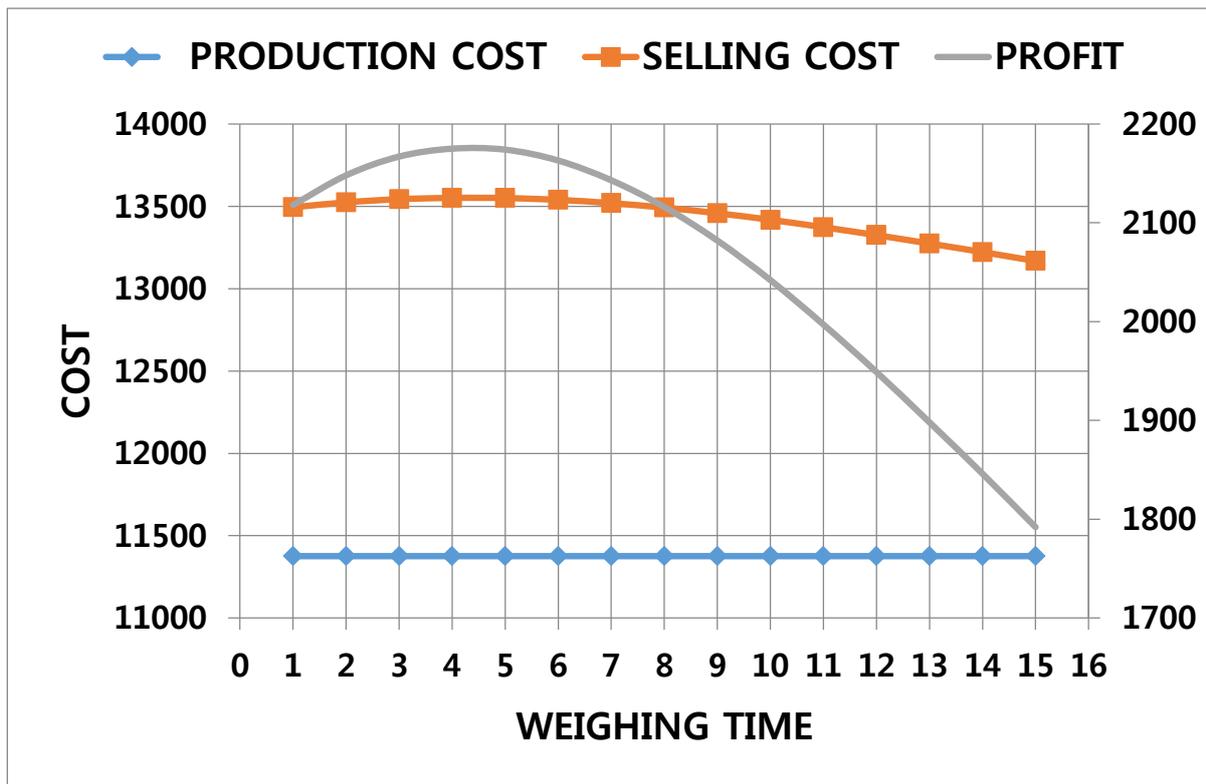


Fig 8: Production Cost, Selling Cost and Profit on Weighing Sensitivity Analysis

It is observed that the time for weighing of the materials for production was varied and the results of the number of custards for the various sizes as well as the cost of production, selling cost and the profit obtained from the simulator were noted in the 3-D column chart. It is seen that the minimum cost of production is constant since the same quantity of material is used in the batch production. The profit made was highest when the weighing time is 4 minutes, hence the weighing operation should be around this time for there to be optimum production and maximum profit.

3.8 Sealing Time Sensitivity Analysis Results

Simulation runs were carried out with the time of sealing varied from 1 to 10, while other input parameters were kept constant. The Dataset for mixing time sensitivity analysis that were kept constant during the simulation run are: [premixing = 20mins, mixing = 20mins, packaging = 3mins, bagging = 3mins, weight of large custard = 1.6kg, weight of medium custard = 1.2kg, weight of small custard = 0.33kg].

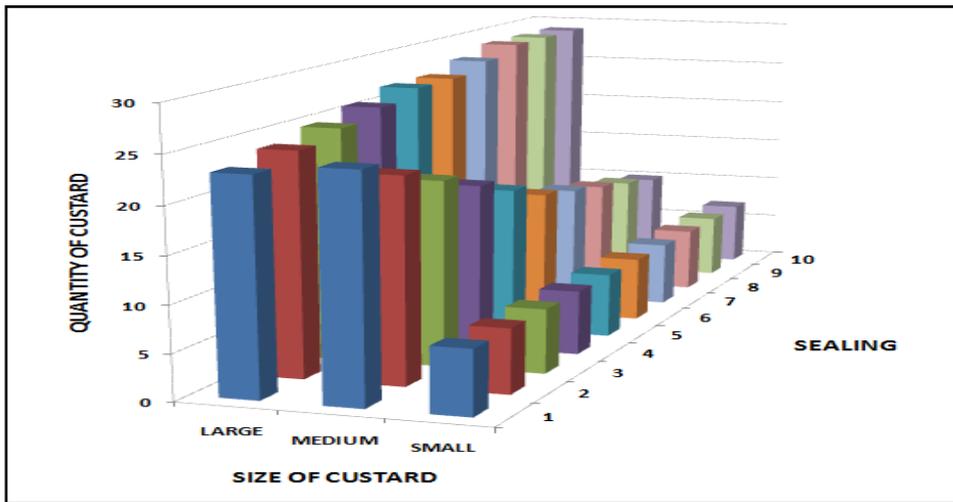


Fig 9: Variation of Sealing Time and Effects on Quantity of Various Sizes of Custard

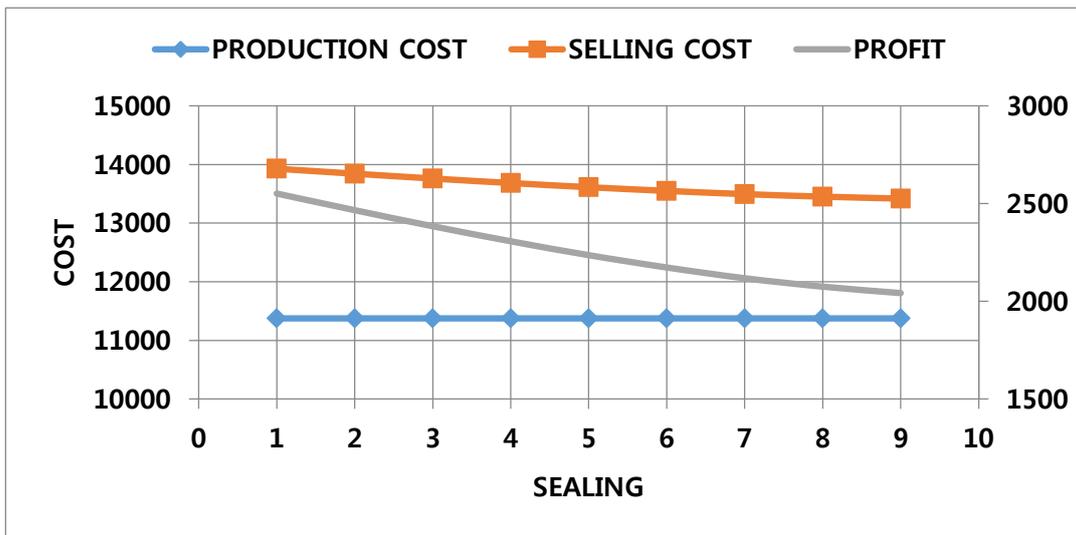


Fig 10: Production Cost, Selling Cost and Profit on Sealing Sensitivity Analysis

It is observed that the time for sealing of the materials for production was varied and the results of the number of custards for the various sizes as well as the cost of production, selling cost and the profit obtained from the simulator were noted as shown in Fig 9 and Fig 10, the production output was highest when the sealing time was least and this implies that the lesser the time spent on this operation, the greater the input of production. Besides, the production cost is constant while the highest profit was made when the time used for sealing was least as depicted from the secondary axis of

Fig. 10. So for there to be maximum yield, there has to be less time spent on the sealing operation.

3.9 Packaging Time Sensitivity Analysis Results

Simulation runs were carried out with the time of weighing varied from 1 to 15, while other input parameters were kept constant. The Dataset for mixing time sensitivity analysis that were kept constant during the simulation run are: [premixing = 20mins, mixing = 20mins, weighing = 3mins, sealing = 3mins, bagging = 3mins, weight of large custard = 1.6kg, weight of medium custard = 1.2kg, weight of small custard = 0.33kg].

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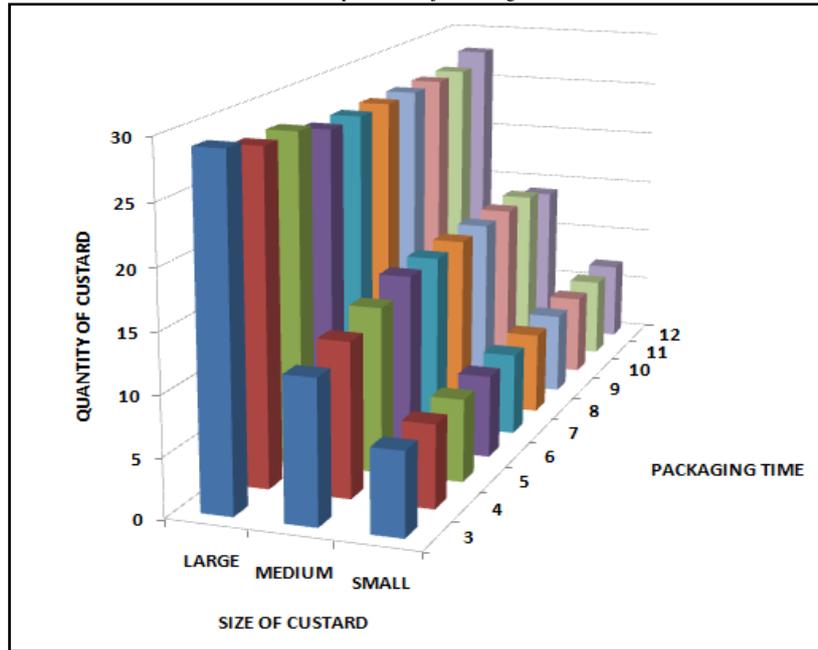


Fig 11: Variation of Packaging Time and Effects on Quantity of Various Sizes of Custard.

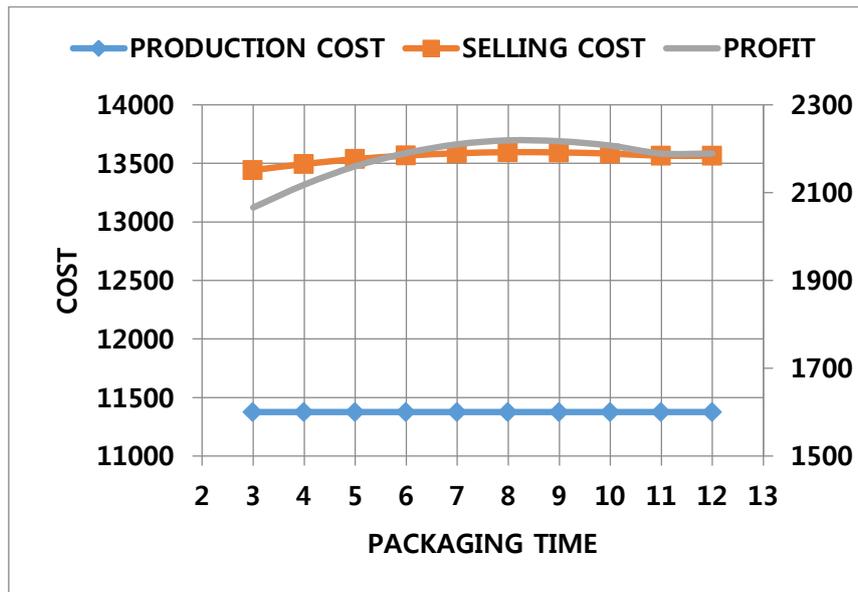


Fig 12: Production Cost, Selling Cost and Profit on Packaging Sensitivity Analysis

3.10 Bagging Time Sensitivity Analysis Results

Simulation runs were carried out with the time of bagging varied from 1 to 10, while other input parameters were kept constant. The Dataset for bagging time sensitivity analysis that were kept constant during

the simulation run are: [premixing = 20mins, mixing = 20mins, weighing = 3mins, sealing = 3mins, packaging = 3mins, weight of large custard = 1.6kg, weight of medium custard = 1.2kg, weight of small custard = 0.33kg].

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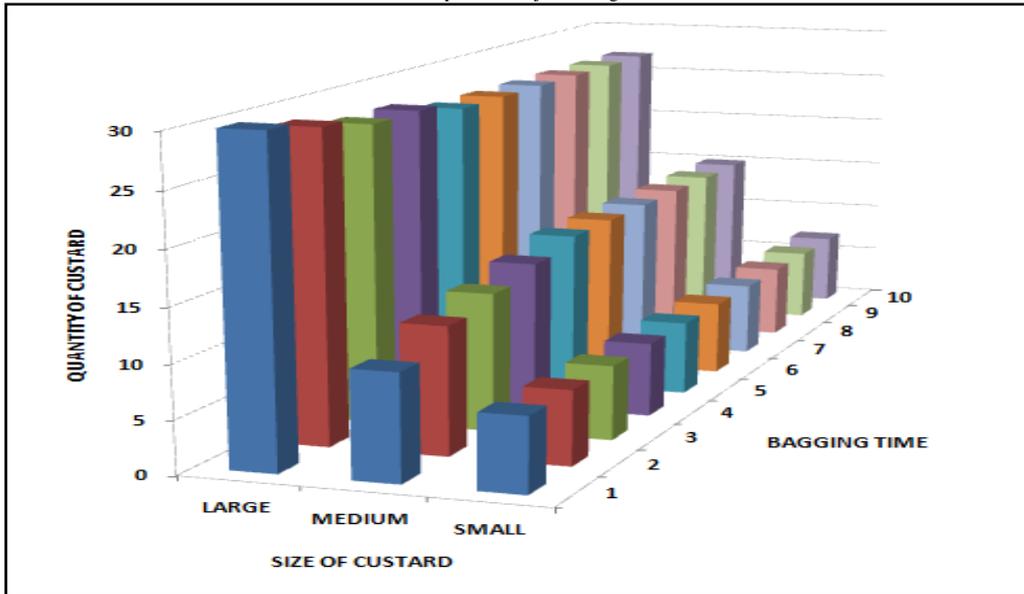


Fig 13: Variation of Bagging Time and Effects on Quantity of Various Sizes of Custard

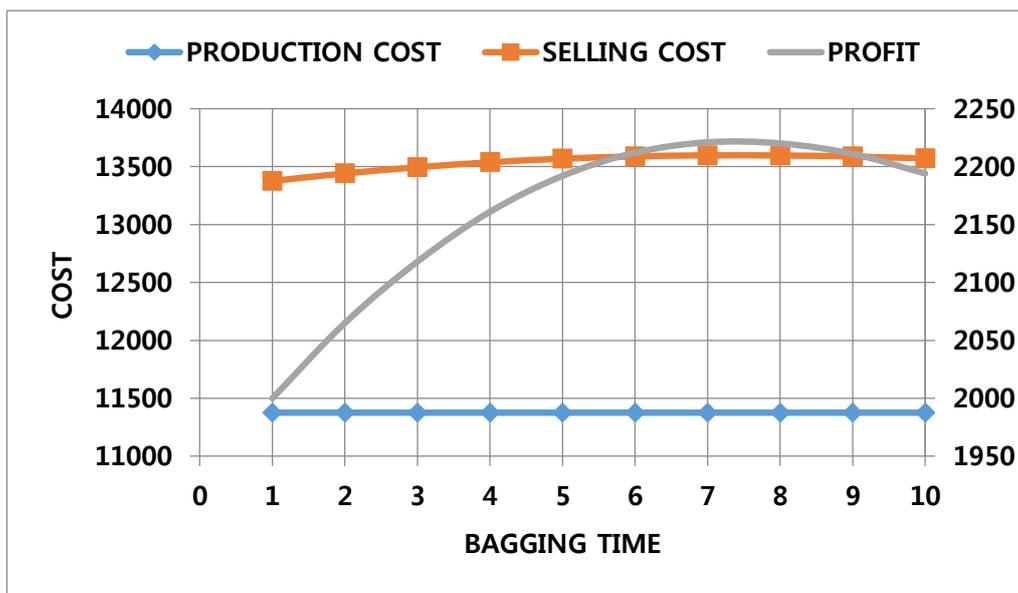


Fig 14: Production Cost, Selling Cost and Profit on Bagging Sensitivity Analysis

4. CONCLUSION

This study has carried out sensitivity analysis of process parameters on the optimal mix and profit margin in custard industries. The results were obtained by performing a simulation using simplex optimization. The designed simulator using a MATLAB and Graphic Users Integrated Development Environment (GUIDE) is capable of detecting, solving and controlling the problems of optimization in custard production industries. The model will help custard producing industries to estimate the quantity of custard powder to be produced, in order to

maximize profit and minimize production cost. The sensitivity analysis from this study will serve as a pedagogical tool of system changes to parameter variations. Beyond custard industries, other production outfits having similar problems can adapt to the models of this study to optimize their production processes.

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REFERENCES

- [1]. Azadivar, F. and Lee, Y. H. (1988). Optimization of discrete variable stochastic systems by computer simulation. *Mathe. Comput. Simulation*, 30: 331-345.
- [2]. Carson, Y. and A. Maria, 1997. Simulation optimization: Methods and applications. Proceedings of the 1997 Winter Simulation Conference, Dec. 7-10, Atlanta, GA, USA., pp: 118-126.
- [3]. Chong P.P., Chen Y.S., Chen C. H. (2001). IT induction in the food service industry. *Industrial Management Data System* 101(1): 13-20.
- [4]. Efstratiadis M. M., Karirti A. C., Arvanitoyannis I. S. (2000). Implementation of ISO 9000 to the food industry: An overview. *International Journal Food Science Nutrition*, 51(6): 459-473.
- [5]. Fu, M. and Healy, K., (1992). Simulation optimization of (SS) inventory systems. Proceedings of the 24th Conference on Winter Simulation, Arlington, VA, USA., Dec. 13-16, ACM Press, pp: 506-514.
- [6]. Hall, J.D., Bowden, R. O. and Usher, J. M. (1996). Using evolution strategies and simulation to optimize a pull production system. *J. Mater. Process. Technol.*, 61: 47-52.
- [7]. Henschion M, McIntyre B (2005). Market access and competitiveness issues for food SMEs in Europe's lagging rural regions (LRRs). *Br. Food J.*, 107(6): 404-422.
- [8]. Knowles G, Johnson M, Warwood S (2004). Medicated sweet variability: a six sigma application at a UK food manufacturer. *The TQM Mag.*, 16(4): 284-292.
- [9]. Lutz, C.M., (1995). Determination of buffer size and location in scheduling systems buffer management, inventory profiles. Ph.D. Thesis, University of Georgia.
- [10]. Okolie, P. C. Okafor, E. A. and Chinwuko, E. C. (2010) Optimal Production Mix for Bread Industries: A Case Study of Selected Bakery Industries in Eastern Nigeria. *Journal of Engineering and Applied Sciences* 5(6) 403-412.
- [11]. Okonkwo U. C. and Obaseki E. (2011) Development of a Stochastic Simulator of (S,S-1) Inventory Policy Differentiated by Demand Lead time through Rationing, *International Journal of Academic Research* 3(1): 81 – 92.
- [12]. Spiegel MVD, Luning PA, De Bore WJ, Ziggers GW, Jongen WMF (2006). Measuring Effectiveness of food quality management in the bakery sector. *Total Qual. Manage.*, 17(6): 691-708.
- [13]. USOCDD (US Overseas Cooperative Development Council) (2007). Cooperatives: Pathways to Economic, Democratic and Social development in the Global Economy, US Overseas Cooperative Development Council, available at: www.coopdevelopmentcenter.coop/OCDC/CoopPathwaysReport.