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Robust Design via NSGA-II with Latin Hypercube Sampling Applied to Structural Beams

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ABSTRACT

In structural beam design, the cross-sectional area is minimized not only to minimize the cost of production but also the weight of the beam. This is subject to constraints on maximum bending stress, maximum beam deflection, and bounds on the dimensions of the beam. Furthermore, the beam design also considers uncertainties in the manufacturing of the beams that may be caused by factors such as measurement precision errors. This is a case of robust optimization problem – solutions are sampled around the neighborhood of a normal solution and the mean and variance of this sample are taken as objective functions. In this study, robust optimization is presented using an elitist Non dominated Sorting Genetic Algorithm (NSGA-II), which does not involve a-priori weights on the objective functions as commonly done in practice. The solutions are evaluated using a penalized function of the cross-sectional area – constraint violation is penalized via static exterior penalty method. On the other hand, the uncertainties are generated using Latin Hypercube sampling instead of the usual Monte Carlo method. The robust solution is then taken from the optimum Pareto front of solutions based on the design priority of the manufacturer. It is found that the solution to the structural beam design without uncertainties tends to yield maximum bending stress. Thus, taking uncertainties into account penalizes the cross-sectional area of the solution leading to large values of average penalized objective function. Robust design on all three beams considered yields an increase of around 10 cm^2 in their cross-sectional area but with reduced bending stress values. Remarkably, the robust design with Latin Hypercube sampling requires fewer samples in obtaining robust optimum than with Monte Carlo method.

Keywords: *evolutionary algorithms, NSGA-II, optimization, robust design*

1. INTRODUCTION

In several applications, there is often a trade-off among various factors to minimize. While optimization of these factors must be done simultaneously, often conflicts arise among some of these factors. Such problem is an example of a multi-objective optimization (MOO), which can be written without loss of generality as

$$\min_{x\in S}\left(f_1(x), f_2(x), \dots, f_p(x)\right) \tag{1}$$

which may or may not be subjected to some constraints. The former is called constrained optimization while the latter is called unconstrained optimization. The functions $\{f_i(x)\}$ are known as the objective functions.

A number of optimization methods are available and oftentimes specific to the type of optimization problem. Deterministic optimization methods include steepest descent, Newton, and quasi-Newton methods. Such optimizers are often approximate solutions and may require regularity conditions on the objective functions. On the other hand, stochastic optimization methods such as the Genetic Algorithm (GA) utilizes the idea of Darwin's Theory of Natural Selection that evolution will yield a population of best-fit solutions [1]. Hybrid methods of these types of optimization were also developed [2, 3, 4].

In the case of MOO, the conventional method is to minimize a weighted sum of the objective functions. The weights are assigned according to priority given to each of the objective functions. The problem of having *a priori* weights is eliminated by getting the set of non-dominated solutions defining the optimum Pareto front and from which a preferred solution is decided according to a prioritized objective. We say that if $\boldsymbol{x}$ and $\boldsymbol{x}^*$ are two solutions of the MOO, then $\boldsymbol{x}^*$ dominates $\boldsymbol{x}$ if and only if

$$f_i(x^*) \le f_i(x) \quad i = 1, 2, \dots, p \tag{2.1}$$

and that there is at least one objective function f_j such that

$$f_j(x^*) < f_j(x) \tag{2.2}$$

Multi-objective GA (MOGA) uses this concept of dominance to yield a subpopulation of non-dominated solutions comprising the optimum Pareto front of solutions.

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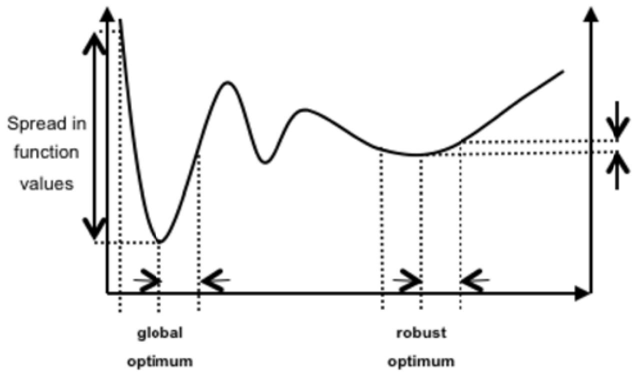

Fig 1: Global optimum versus robust optimum.

Meanwhile, a number of optimization problems especially in the field of engineering are affected by random and uncontrollable factors such as raw materials, machine settings, temperature variations, etc. Hence, uncertainties must be considered and a robust design is needed to mitigate the effect of these uncertainties. In this respect, the optimization problem becomes a robust optimization problem [5, 6]. Here, the goal is not necessarily the solution that gives the global optimum but the robust optimum that provides allowance in the design accuracy of the parameters as illustrated in Figure 1.

Robust optimization via GA is commonly done by giving priority on either the mean or variance or a weighted sum of the two [2, 7]. In this paper, we present a robust optimization problem applied to three structural beam designs namely I-section, T-section, and C-section beam designs, using an elitist Non-dominated Sorting GA (NSGA-II) with Latin Hupercube sampling. We present the I-Beam design problem as a robust optimization problem in the following section followed by the implementation of NSGA-II as the optimizer. A similar implementation for the T-beam and C-beam designs can be constructed in the same manner. The results for the beam designs will be discussed and a summary of the study will be made hereinafter.

2. ROBUST STRUCTURAL DESIGN

2.1 I-beam Design

The I-beam design [8] consists of a simply-supported beam under maximum axial and transverse loads. Figure 2 shows the side view and the cross-sectional area of the beam, labeled with the design parameters. Table 1 gives the structural parameters and the support and loading conditions for the I-beam.

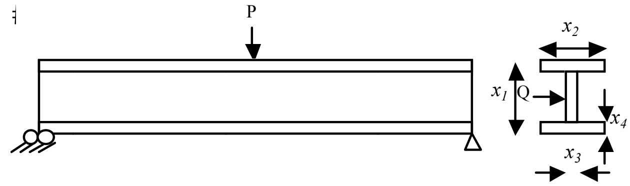

Fig 2: The parameters for the I-beam design. Left: Longitudinal section of the beam. Right: Cross-section of the beam.

Table 1: Specifications for the I-beam design

Allowable bending stress	16 kN/cm^2
Maximum bending forces	P = 600 kN
	Q = 50 kN
Young's modulus of elasticity	2x10^4 kN/cm^2
Length of the beam	200 cm

The main objective of the I-beam optimization [2] is to minimize the cross sectional area given by

$$f(x) = 2x_2x_4 + x_3(x_1 - 2x_4) \tag{3}$$

subject to the following constraints:

$$g_1(x) = \frac{180000x_1}{x_3\left(x_1 - 2x_4\right)^3 + 2x_2x_4\left(4x_4^2 + 3x_1\left(x_1 - 2x_4\right)\right)} + \frac{15000x_2}{\left(x_1 - 2x_4\right)x_3^3 + 2x_2^3x_4} \le 16 \tag{4}$$

$$g_2(x) = \frac{5000}{\frac{1}{12}x_3\left(x_1 - 2x_4\right)^3 + \frac{2}{12}x_2x_4^3 + \frac{1}{2}x_2x_4\left(x_1 - x_4\right)^2} \le 0.1 \tag{5}$$

The constraint functions $g_1(x)$ and $g_2(x)$ are the bending stress and vertical deflection on the I-beam, respectively. The beam design is also constrained on its bounds:

$$\begin{aligned} 10 &\le x_1 \le 80 \\ 10 &\le x_2 \le 50 \\ 0.9 &\le x_3, x_4 \le 5 \\ 2x_4 &\le x_1 \end{aligned} \tag{6}$$

We now consider the design of the I-beam with dimensional uncertainty. Sources of this uncertainty may include but are not limited to manufacturing limitations or preference of low grade, less expensive materials with high variability. The dimensions of the beam are accurate to some precision and the uncertainty is assumed to follow a certain distribution. For our structural beam design, we take suitable error tolerances based on Steel Handbook [9], i.e. $\Delta x = 0.5$ for the first and second dimensions and $\Delta x = 0.05$ for the third and fourth dimensions.

2.2 Robust Design via NSGA-II

In this section, we briefly discuss the principles of Genetic Algorithm (GA) and then we proceed with the technical aspects of NSGA-II.

As mentioned earlier, GA is based from the Theory of Natural Selection, which starts from an initial pool of individuals (population of initial solutions). A fitness value is associated to each of these individuals a number that is inversely proportional to the objective function to be minimized. The population is then evolved at each generation through three Darwinian principles, namely: selection, crossover, and mutation.

Individuals are selected through a non-uniform roulette wheel method according to their fitness value. With a probability p_c, two selected parent individuals breed two new individuals from a uniform random combination of the parents. Finally, each of the new generation individuals mutates into a new individual with a probability p_m via a non-uniform mutation. This mutation process is such that the strength of mutation decreases with generation number. A 1-elitist GA includes the preservation of the best individual in the previous generation if none of the current population surpasses it.

Two or more objective functions are considered in a multi-objective GA (MOGA). It utilizes the dominance criterion (2) to identify the non-dominated individuals that will comprise the Pareto front at each generation. Individuals here are ranked according to the number of dominated individuals. As a consequence, fitness sharing may lead to a premature convergence to a sub-optimum Pareto front. II.

NSGA [10, 11, 12] is a variant of MOGA in which, successive Pareto fronts are constructed from the set of dominated individuals. Hence, individuals are ranked by front and a niche is created to maintain diversity in the subpopulation of individuals in the same front. This is done by utilizing a sharing criterion, σ_{share} given by:

$$Sh(d_{ij}) = \max\left(0, 1 - \frac{d_{ij}}{\sigma_{share}}\right) \tag{7}$$

where d_{ij} is the phenotypic distance between two individuals x_i and x_j in the same front. Finally, a new dummy fitness f_i is assigned to each individual x_i of a current front F_p according to:

$$\begin{aligned} f_i &= f_p \sum_{j \in F_p} Sh(d_{ij}) \\ f_p &= 1 + \max_{j \in F_{p-1}} f_j \end{aligned} \tag{8}$$

where f_p is the dummy fitness of the current front.

We now implement NSGA in the robust design of the I-beam. The main objective of the I-beam optimization given by (3) is maintained but instead of subjecting it to constraints (4)-(6), these constraints are being incorporated to the main objective function by a static exterior penalty function with k as the strength of penalization. Thus, the objective function of the I-beam design is condensed to:

$$\begin{aligned} J(x) = f(x) + k\Big(&\max(0, g_1(x) - 16)^2 + \max(0, g_2(x) - 0.1)^2 \\ &+ \max(0, 10 - x_1)^2 + \max(0, 10 - x_2)^2 + \max(0, 9 - 10x_3)^2 \\ &+ \max(0, 9 - 10x_4)^2 + \max(0, 2x_4 - x_1)^2 \Big) \end{aligned} \tag{9}$$

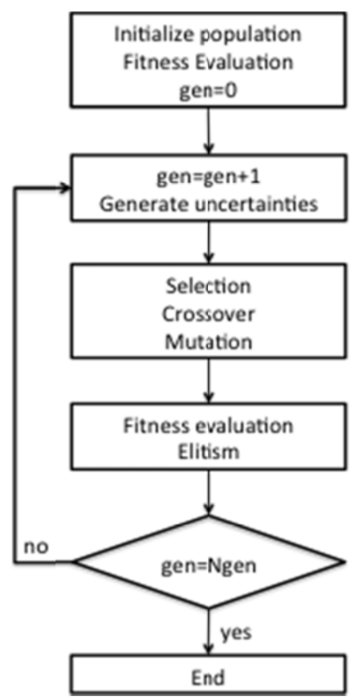

Fig 3: Flowchart for robust optimization using NSGA-II.

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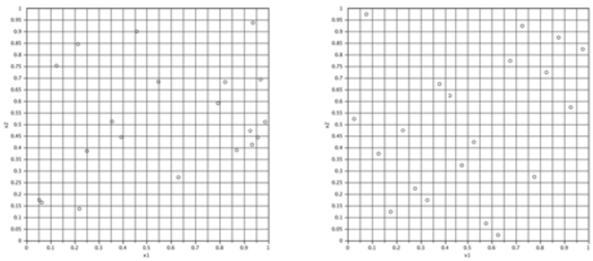

Fig 4: Monte Carlo sampling versus Latin Hypercube sampling of 20 points in two dimensions.

Robust optimization is then considered by modeling uncertainties as follows: At each generation, N_{sample} uncertainties $\{\xi_k\}$ are generated on [-Δx, Δx]. This way, the sampling is the same for every individual at each generation compared when done per individual. This is also to be able to compare all the solutions with the same sample of uncertainties and consequently reduces the cost of generating uncertainties for each individual. Then for each individual x_i, $x_i + \xi_k$ is evaluated using (9) for k =1, 2, … N_{sample}. Finally, the mean and variance of the function values, averaged over ten samplings, are taken as the two objective functions for our robust optimization. Strict dominance is used to identify the successive fronts and the σ_{share} value is taken as the norm of $\Delta x = [0.5\ 0.5\ 0.05\ 0.05]^T$ which is approximately 0.7106. Finally, since the dummy fitness is dependent on the uncertainties sampled at each generation, the elite solution is preserved only when it is more robust than the best individual in the current population. This is done by comparing the norms of the objective function vector (mean and variance) for the elite solution and the current best individual in the population. The flowchart for this design using NSGA-II is illustrated in Figure 3.

In this study, the uncertainties are generated using Latin hypercube sampling (LHS), in contrast to traditional Monte Carlo. Figure 4 shows the difference of the two samplings. LHS is an extension of stratification in multiple dimensions. It treats all coordinates equally and avoids the exponential growth in sample size resulting from full stratification by stratifying only the one-dimensional marginal of a multidimensional joint distribution [13].

3. RESULTS AND DISCUSSION

Global optimization is first performed for the beam design alone without uncertainties. The penalty strength is set to k=10^5 (high penalty on constraints) so that the solution is kept inside the admissible set. The optimum dimensions of the beams found are shown in Table 2 while the objective and constraint function values are shown in Table 3. We can see that the solutions are well within the constraints on dimension bound and beam deflection whereas the bending stress values are approaching the maximum allowable value for all three beams. These solutions are then tested for sensitivity by evaluating 10^5 samples around their neighborhood. One can expect that the objective function will be heavily penalized due to constraint violation on bending stress. The last column of Table 3 verifies this intuition. It is also remarkable that even if the global solution for the T-beam also has a beam deflection near the maximum allowable value, it has the least average penalized cross-section. Note that the value of a penalized objective function depends on the penalty strength k, which is the number of units increase in the objective function per unit of constraint violated.

Table 2: Global solutions for beam designs without uncertainties

Beam	x_1 cm	x_2 cm	x_3 cm	x_4 cm
I	60.67171	40.90100	0.91348	0.91171
T	74.325199	49.37504	0.94409	1.84095
C	62.109589	34.46753	0.90809	0.96229

Table 3: Objective and constraint function values of global solutions for beam designs without uncertainties

Beam	Cross-sectional Area cm^2	Bending Stress kN/cm^2	Beam Deflection cm	Average Penalized Cross-sectional Area cm^2
I	128.33621	15.99997	0.06090	1.472E+04
T	159.32855	15.99998	0.09199	3.960E+03
C	120.98924	15.99998	0.06369	2.055E+04

Robust design via NSGA-II is then implemented with uncertainties now introduced in the design of the structural beams. For each generation, we keep track of the most robust solution found via NSGA-II with the least fitness value determined by the front and the niche of the solution. We note at this point that most of the individuals in the final optimum Pareto front, such as in Figure 5 are actually solutions that only differ starting in the third decimal digit.

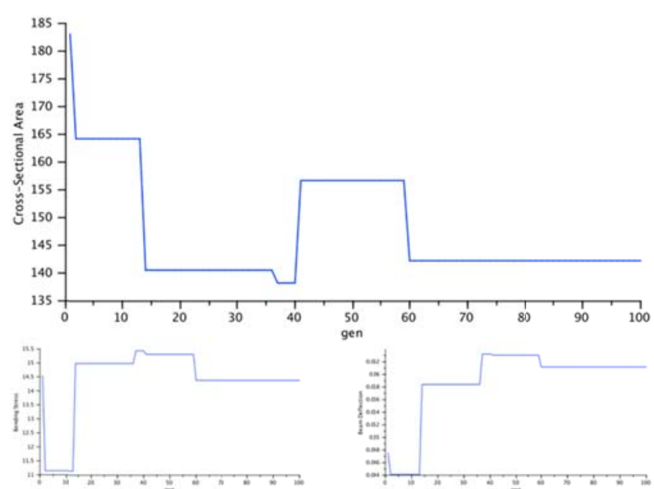

Fig 5: Convergence to a robust solution for the I-beam design. The graphs show the cross-sectional area (upper),

bending stress (lower left), and beam deflection (lower right) of the most robust solution for each generation.

Unlike global optimization, we don't expect a steady decrease in the objective function for our robust optimization. Figure 6, for example, shows the evolution of the most robust solution for the I-beam design. Our robust optimizer may find a solution with higher objective and/or constraint values than a previous generation but with less mean and variance when uncertainties are considered.

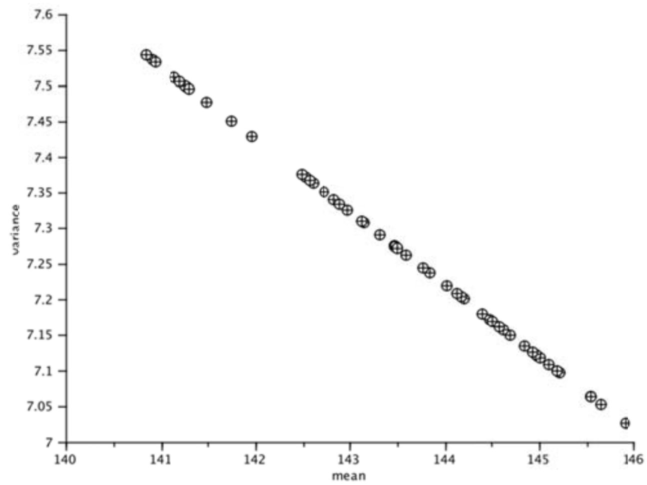

Fig 6: Optimum Pareto front for I-beam design via NSGA-II with Latin Hypercube sampling.

Tables 4 to 6 summarize the best results out of ten independent runs for the robust designs of I, T, and C beams respectively using several number of sampled uncertainties generated via LHS. Increasing the number of samples does not generally show increase in the cross-sectional area of the beams. For the I-beam design, a robust solution with minimum cross-sectional area is found using 20 samples via LHS while 10 and 40 samples for the T-beam and C-beam designs, respectively. The dimensions of the beams corresponding to these robust solutions are given in Table 7. For comparison against the global solutions disregarding uncertainties, Table 8 gives the objective and constraint function values for the solutions found from our robust design.

Table 4: Average objective and constraint function values for the I-beam design with uncertainties.

Number of Samples	Cross-sectional Area cm^2	Bending Stress kN/cm^2	Beam Deflection cm
10	146.3230	14.8934	0.0490
20	140.6798	14.4200	0.0614
40	148.0608	15.1234	0.0478
50	154.7670	14.9611	0.0732
80	153.8994	14.1094	0.0592

Table 5: Average objective and constraint function values for the T-beam design with uncertainties

Number of Samples	Cross-sectional Area cm^2	Bending Stress kN/cm^2	Beam Deflection cm
10	168.4738	15.1097	0.0948
20	180.8420	15.4350	0.0728
40	196.4443	14.0186	0.0842
50	190.3637	15.0685	0.0771
80	184.0177	14.6265	0.0880

Table 6: Average objective and constraint function values for the C-beam design with uncertainties.

Number of Samples	Cross-sectional Area cm^2	Bending Stress kN/cm^2	Beam Deflection cm
10	144.3565	15.3710	0.0683
20	145.6660	15.4060	0.0691
40	132.7499	15.0343	0.0454
50	145.3094	15.2036	0.0733
80	149.0795	15.0085	0.0618

Table 7: Robust solutions for beam designs with uncertainties

Beam	x_1 cm	x_2 cm	x_3 cm	x_4 cm
I	56.88822	43.60033	0.97582	0.99904
T	71.74836	48.10488	0.98181	2.08031
C	73.01147	28.68687	0.99194	1.08914

Table 8: Minimum objective and constraint function values of robust solutions for beam designs with uncertainties.

Beam	Cross-sectional Area cm^2	Bending Stress kN/cm^2	Beam Deflection cm	Average Penalized Cross-sectional Area cm^2
I	140.67981	14.42003	0.06136	140.67981
T	168.47380	15.10968	0.09484	168.47380
C	132.74991	15.03426	0.04537	132.74991

Table 9: Minimum objective and constraint function values of robust solutions for I-beam design using Monte Carlo (MC) versus LHS.

Uncertainty Sampling	Cross-sectional Area cm^2	Bending Stress kN/cm^2	Beam Deflection cm
MC	149.18660	14.95580	0.07106
LHS	140.67981	14.42003	0.06136

An increase of around 10 cm^2 in the cross-sectional area is found for all three beams. The values of the average penalized cross-sectional area suggest that uncertainties do not incur significant constraint violations. This can be explained by lower bending stress values (by around 1 kN/cm^2) for all three beam design solutions especially the I-beam. Moreover, the C-beam solution has much lower beam deflection (by around 0.02 cm). We also underscore in this study that the design solutions found using LHS to generate the uncertainties in the beam dimensions have lower cross-sectional area, bending stress, and beam deflection compared to the solutions found using Monte Carlo sampling. Table 9 compares the values found for the I-beam design. This suggests that LHS can be more appropriate than Monte Carlo for robust design, especially with less number of samples implying less number of function evaluations per generation in the NSGA-II.

4. CONCLUSION

NSGA-II was utilized for robust design applied to structural beams. The design involves static exterior penalization of the cross-sectional area, using the mean and the variance as the objective functions in the NSGA-II, and LHS in generating the uncertainties for normal solutions. The robust solutions found for all three beam designs (I, T, and C) incurred an increase of around 10 cm^2 in cross sectional area but were found with lower bending stress values by around 1 kN/cm^2 compared to the global solutions whose bending stress values approach the maximum allowable value of 16 kN/cm^2. The robust solutions were also found to have lower beam deflection especially the C-beam, reduce by around 0.02 cm. The robust design prototype presented in this study is very likely applicable for other robust design problems in structural, mechanical, transportation, and electrical engineering.

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