Compressed Spectrum Sensing in Cognitive Radio Network Based on Measurement Matrix

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ABSTRACT

Compressive sampling (CS), or compressive sensing, has the ability for reconstructing a sparse signal with small number of measurements. There are some applications like spectrum sensing in cognitive radio which not necessarily need a perfect reconstruction. Consequently in this application, toward the decrement of high signal acquisition costs in wideband system, CS methods have been used for spectrum sensing. New developments in CS have presented a new way toward the reconstruction of the original signal by using minimum number of observations. In this paper, we present a novel method in which CS is employed for compressing spectrum sensing in CRNs. Also, we include the explanation for showing that how CS utilization actually can attain the advantage of sampling and computational complexity reduction at a same time. For simulating the compressive sensing application in Cognitive Radio network the measurement matrix made by some random numbers is multiplied in the spectrum which is occupied by users. The mentioned measurement matrix is chosen with a procedure in which by using an optimization technique the sparse spectrum can be precisely recovered. By using an available multiple optimization technique the spectrum can be reconstructed by small number of samples. MATLAB software is used for the simulation of the algorithm. A reliable Spectrum sensing, even in low SNR, and small number of samples, is confirmed by results of the simulation. These results demonstrate that this method can lead to a faster measuring range in comparison with other existing approaches.

Keywords: Cognitive radio, compressive sampling, spectrum sensing, measurement matrix

1. INTRODUCTION

Considering the fast growth of radio access equipment, spectrum overcrowding has risen to be a new challenge. However, a Federal Communication Commission survey shows that a relatively large percentage of the assigned spectrum is not used most of time [1]. In order to elucidate this confliction between radio resource shortages versus underutilization, cognitive radio technology has been introduced with which quick sensing and opportunistically the unoccupied spectrum utilization without interfering with any primary user (PU) is possible [2]. The primary task in applications related to CR is performing spectrum sensing to identify available spectrum.

By utilizing blindly detection of the spectrum holes, the compressed sensing theory can be used to reduce the dynamic spectrum sensing issue [3], [4]. The rudimentary idea in CS is sampling compressible signals at a lower rate in comparison with the Nyquist rate and later reconstructing signals with compressed measurements [3].

Number of measurements in CS method is much fewer compared to Nyquist sampling. As a result this leads to a noticeable sampling rates reduction. Consequently, the analog-to-digital converter resource necessities are reduced considerably, which has a significant value in wideband communication systems [5]. Up to this date, a lot of techniques based on CS have been presented [3], [5].

There is a lot of hope for the novel CS theory that it be able to reduce the sampling rate and decrease computational costs at a CR node for compressible signals [6].

A number of CS methods have been proposed in order to detect wideband PU frequency of occupancy at a smaller range of sampling rate [7], [8]. Toward the determination of the appropriate sampling rate, all of these studies assume that the order of sparsity in the underutilized spectrum is identified precisely. Lately, many researchers have been concentrated on random projection. In [9] a method of cooperative compressed spectrum sensing is presented, in where with a Gaussian procedure model, the reconstruction of compressed spectrum is displayed. In [10] the dynamic resource allocation problem in CRNs has been scrutinized. In this reference, for detection of occupied spectral bands with compressed measurements, multiple CS-based techniques are utilized. In current CRNs, the CS with the goal of the sampling rates reduction in the acquisition of compressible signals is utilized to lighten the sampling bottleneck [11], [12]. There are three key technical challenges in the spectrum sensing: 1) a direct relationship between the augmentation of the sampling rate and cost of signal processing, 2) challenges and difficulties in the radio front-end design, and 3) in older ways of spectrum estimation, high speed DSP is used which operates at or above the Nyquist rate. This may cause failure in perfect signal reconstruction since the requirement on spectrum sensing timing windows are high [3].

In this paper, a measurement matrix based on compressive sampling is presented to compress the Spectrum sensing in CRNs. The aforementioned approach, utilizes CS-based spectrum sensing which with a sampling rate lower than Nyquist rate, can sense the bands which...
are blind occupied by using a measurement matrix. The compressive sensing algorithm output is the observation vector. By using different optimization techniques like $l_1$-norm optimization, spectrum is reconstructed from a small number of samples. In order to compress the signal with a rate lower than that of the Nyquist and reconstruct it by a multiple optimization techniques provided that no important information is lost, MATLAB simulations were performed.

In the rest of the paper, Related Work is studied in next section. Section 3 the Compressed spectrum sensing model is given and explains the functionality of each part in the model; Section 4 provides Analysis and Design of the proposed compressed spectrum sensing model; Performance Simulations are given in Section 5; Finally Section 6 draws the conclusion.

2. RELATED WORK

The compressive sensing was improved by Candes et al [13] and Donoho [14] in 2005. These methods include captivating random signal projections and then, by using one of the available optimization techniques, recovering the signal from a low number of measurements. In an older sampling theory, a signal is sampled by using the Nyquist rate, while it can be sampled with a rate lower than Nyquist by using compressive sensing. The reason which makes this possible is because of the signal transformation into a new domain which posses a sparse representation. After doing this, the signal will be reconstructed from a small number of samples with an optimization technique. Although CS is a capable technique but it does not let the CR to sample at low rates without imposing a high cost; because the lower rates increase the computation complexity and so computation cost.

It is possible to classify the CS techniques spectrum sensing into two categories: (1) Convex relaxing-based methods, like basis pursuit [15], [16] and LASSO and Dantzig Selector [17]; (2) Greedy algorithm based methods, like matched pursuit (MP) [18]. Lately, several MP-based methods have been introduced like orthogonal matched pursuit [19], regularized orthogonal matched pursuit [20] and compressive sampling matching pursuit [21]. In fact, the aforementioned approach can achieve better reconstruction precision; however it may result in high computation costs. The greedy algorithm-based methods have the advantage of having less computational complex; nevertheless reconstruction precision is lower in comparison with the convex programming. Compressive sensing has some different application in wireless communications. In [22], a two-step compressed spectrum sensing method with the purpose of the proficient wideband sensing is presented.

In [23], by bypassing signal reconstruction, a Bayesian compressive sensing frame decreases the sampling necessity and computational complication. In [24], in order to gather spatial variety against wireless fading, manifold CRs team up and result in an establishment of agreement among local spectral estimates with running a program for decentralized agreement optimization. In [25], in order to scan wide spectra compressive sampling is performed at confined CRs then collected measurements of manifold CR detectors are combined to gather spatial variety. This enhances the quality of detection specially under fading channels.

3. COMPRESSED SPECTRUM SENSING MODEL

3.1 Signal Representation and Sparsity

Signal representation and sparsity is important in compressive sensing. Suppose that $x \in \mathbb{R}^d$ represent a real signal, and the signal $x$ is sparse in the orthogonal basis $\psi = \{\psi_1, \psi_2, \psi_3, ..., \psi_N\}$. Here $N$ is the signal length, by this assumptions $x$ can be represented by combining $K$ basis functions linearly:

$$X = \sum_{i=1}^{K} \theta_{ni} \psi_{ni}$$

(1)

Here $\psi_{ni} \in \psi$, $Ni \in [1, 2, 3, ..., N]$. Again assume that $\theta = [\theta_1, \theta_2, \theta_3, ..., \theta_N]^T$ be the coefficients vector for the signal $x$ in $\psi$. The signal $x$ random measurement can be shown as following:

$$y = \phi \theta, \phi: M \times N, \ K < M \ll M$$

(2)

Where $y$ is the signal $x$ measurement vector, $\phi$ is the uniform random measurement matrix, $\theta$ is the coefficients vector and $M = cK$ ($c < 1$) is the number of measurements which are needed for perfect reconstruction. If we receive all of the entries of $\phi$ from a Gaussian distribution, we have the ability to reconstruct the signal $x$ perfectly with high probability for achieving success provided that the constant ‘c’ is between 3 and 5 [26]. Transform coding is the name of the process which is used to guarantee the sparsity of the signal and is performed in 4 steps [27]

a. Acquire all N-points of signal $x$ by using the Nyquist rate
b. Calculate the whole set of transform coefficients
c. Detect the $K$ largest coefficients then omit the smallest ones
d. Use the measurement matrix to obtain the observation vector of length $M$ by multiplying signal in it.

3.2 Measurement Matrix

In this section, the emphasis is on the representation of the signal with a coherent basis. Consider the Linear measurement procedure that calculates $M < N$ inner products between $x$ and the vectors collection $(\phi_j)_{j=1}^M$ through, $y_j = \langle x, \phi_j \rangle$, in here $j=1, M$ and $\phi_j$ signifies the transpose of $\phi_j$ and $\langle \cdot, \cdot \rangle$ signifies the inner products. Assume an Mx1 vector $y$ for the measurements array $y_j$. In matrix representation, the vector $y$ is acquired with:
\[ y = \Phi x = \Phi \Psi \alpha \]  

(3)

\( \Phi \) represents an \( M \times N \) measurement matrix. In this matrix each row denotes a measurement vector \( \Phi_{ij} \) and \( \alpha \) represents the coefficient vector which has \( K \) none zeroes components. Note that, provided that they are incoherent with fixed basis \( \Psi \) like Gabor, sinusoidal and wavelets some of the measurement matrices have the capability to be used in any special scenario.

The measurement matrix has also an important part in the procedure of recovering the original signal. Two kinds of measurements matrices have the requirements to be used in compressive sensing: The Random measurement matrix and the predefined measurement matrix.

It is important to mention that if signal \( x \), composed of \( N \) samples, is sparse then the real signal is recoverable by using \( M \geq O (K \log (N/K)) > N \) linear projection of \( x \) onto another basis. Moreover, \( x \) can be thoroughly reconstructed with different optimization techniques. Provided that \( \Phi \) is a random matrix, its rows will not be stochastically independent since they are acquired randomly from the same random seed vector. The aforementioned matrix is then transposed and orthogonalized. By doing so, the matrix can represent an orthonormal basis. In a predefined measurement matrix, Dirac and Sine functions are utilized to build the matrix. In the former, the signal is multiplied by Dirac functions which are centered at diverse positions to get the observation vector. Then the signal will be reconstructed by using the \( l_1 \) normalization method as well as the observation vector and the predefined measurement matrix.

Linear programming is indispensable in reconstructing the original signal too. It is a mathematical method which is considered to acquire the best result in a specified mathematical model. Linear programming can be articulated as it comes at follow:

Maximize \( c^T x \)

Subject to \( Ax \leq b \)  

(4)

Here \( x \) denotes the variable which is needed to be specified, \( c \) and \( b \) are coefficients vectors and \( A \) is a matrix of coefficients. The shown function which is needed to be optimized is called the objective function and the equation \( Ax \leq b \) expresses the constraints that should be considered along with the required optimization. Note that, the reconstruction of the signal will depend on the observation vector and the measurement matrix.

### 3.3 Signal Reconstruction in Compressive Sensing

Late progresses in signal theory have revealed that a sparse signal can be a useful tool in different subjects including communications, radar and image processing. Hence, in the compression of the signal, assuming that every signal has the ability to be signified in a sparse form can be helpful. The measurement matrix \( \Phi \) and the measurement vector \( y \) determine whether a signal can be thoroughly restructured or not. It can be concluded from compressive sensing theory that provided that the matrix \( \Phi \Psi \) has the Restricted Isometric Property (RIP) then it becomes possible to recover the \( K \) largest substantial coefficients from a similar size set of \( M = O (K \log (N/K)) \) measurements of \( y \). Subsequently, it will be provided to reconstruct the sparse signal by various optimization techniques like \( l_1 \) norm optimization. The first one is a minimization technique is used to reconstruct the signal:

\[
(P1) \min \|X\|_1 \quad \text{Subject to} \quad y = \Phi x
\]

This method is also called basis pursuit (P1). The objective is discovering the vectors with the smallest \( l_1 \)-norm

\[
\|x\|_1 = \sum_{i=0}^{n} |x_i|
\]

(6)

### 3.4 Compressive Classification

In this section the classification of compressive sensing according to its application is discussed and the effect of compressive sensing and its performance is investigated. It is possible to use the compressive sensing framework in different statistical inference tasks, for instance in finding the solution of detection problems that in these problems CS has the capability to reconstruct the original signal from the limited number of measurements.

- **Classical Detector**

  One of the classifications in compressive sensing is the classical detector. There are two premises relating the signal; whether signal is present in the measurement or not. In the classical Neyman-Pearson (NP) detector a likelihood ratio test is used in which a threshold \( \gamma \) and the sufficient statistic \( t = (y, x) \) are compared against each other. Where \( y \) denotes the measurements, \( x \) signifies the original signal and \( \gamma \) is set in a way that achieving a specified probability of false alarm \( P_F \leq \alpha \) for \( 0 \leq \alpha \leq 1 \) be possible. It is proved that (refer to [29]):

\[
P(\alpha) = Q(Q^{-1}(\alpha) - \sqrt{SNR})
\]

(7)

Here \( Q(.) \) is the reversed form of standard Gaussian cumulative distribution function.

- **Compressed Detector**

  In [30] a theory is explained that can be easily extended provided that the measurements are made with a compressed sampler. In this section, the assumptions shown in below are considered:

\[
H_0: y = \phi n
\]

(8)

\[
H_1: y = \phi (X + n)
\]

(9)
Here $n \sim N(0, \sigma^2)$ is white Gaussian noise with mean zero and a standard variance. The sufficient value is $\hat{\epsilon} \equiv (y, \phi x)$. The probability of false alarm can be approximately specified using the following equation [12]:

$$P(\epsilon) = Q \left( Q^{-1}(\epsilon) - \sqrt{\frac{M}{N}} \sqrt{\text{SNR}} \right) \quad (10)$$

Here $M$ is the number of random projections and $N$ represents the signal sparsity. By comparing the two equations of (2.7) and (2.10) it can be concluded that the detector performance gets worsened if $M$ decreases. Also, the detector performance is influenced by degradation rate of the SNR.

- **Compressive Classifier**

Assume that we have generalized results for a set $X = \{x_i\}$ of signals for a multi-class classifier. The detection rule is determined with $\hat{\mathcal{X}} = \min_{i \in \mathcal{X}} |y - \phi x_i|_2$. The distances are not conserved in this classification, and somehow in comparison with the matrix $\phi$ where the distances are conserved, they are uniformly shrunken. For a signal like $x$, and a typical value of $\epsilon$, with probability of at least $1 - \delta$, the following equation is true for all $x_l$, [30]

$$(1 - \epsilon)\sqrt{\frac{M}{N}} \leq \frac{\|x(x - x_l)\|_2}{\|x(x - x_l)\|_2} \leq (1 + \epsilon)\sqrt{\frac{M}{N}} \quad (11)$$

It can be driven from above equation that according to SNR the effect of noise can be augmented with the transformation.

### 3.5 Optimization Techniques

Signal reconstruction is important in compressive sensing theory that the signal is reconstructed from a small number of measurements. As it was mentioned before precise reconstruction of a signal with an optimization technique is possible. Also, there are a series of papers that discuss theory of signal recovery from incomplete information.

- **$l_1$ Minimization**

Some new papers have developed a theory for recovering a signal from highly incomplete information. These papers show that a sparse vector $x \in \mathbb{R}^n$ can be recovered from a minimum number of linear measurements. The $l_1$ minimization is often utilized to solve the under determined linear equations or sparsely corrupted solution to an over determined equations.

### 4. PROPOSED MODEL DESIGN AND DESCRIPTION

In the remaining of this section to improve the data rates usage of compressive sensing in a cognitive radio network is investigated. The goal in here would be to improve the data rates of current and future generation cognitive radio network. In the presented system the spectrum including PUs is sampled by using compressive sensing with a rate lower that of the Nyquist. In the next step, the compressed spectrum is transmitted over the system and without losing any important information will be reconstructed at the receiver end. In the first phase of the project, in a cognitive radio the spectrum which is occupied by users was modeled. The aforementioned model was then mapped into the discrete frequency domain by utilizing the FFT. In the second phase, prior to applying the compressive sensing to the spectrum signal and in order to remove the coefficients that are not important to the signal a threshold window was applied. The aim of using this threshold is to guarantee that the FFT spectrum is sparse.

In the third phase, the resulted spectrum was multiplied by the measurement matrix. To be able to transmit the output of the procedure by system, it is transformed into a digital spectrum signal by an Analog-to-Digital converter. At the reconstruction, by using the measurement matrix and the observation vector an initial guess was made that is near the main spectrum. At the final stage, using one of the optimization techniques, the spectrum signal was reconstructed from a small number of observations.

The FFT is an algorithm that has simplified Fourier analysis and digital signal processing of digital. The FFT algorithm uses some small tricks and has the ability to calculate the Fourier transform of a discrete signal in $N \log_2(N)$ processes. The following two factors are necessary to be considered while implementing the FFT in MATLAB software:

a. Complex numbers are used in FFT

b. In FFT both of positive and negative frequencies are calculated.

This is the reason that brings difficulties toward the implementation of FFT in compressive sensing. The main problem is that a hard and tedious procedure is needed to apply compressive sensing to a complex number. This process is being investigated at Michigan State University by employing a hybrid compressive sensing model [31].

To solve the previous problems instead of FFT a Discrete Cosine Transform (DCT) is being used. The DCT is the same as DFT in concept except for doing better in the energy concentration into lower order coefficients in comparison with the DFT. Also the DCT possess another great advantage and it is that all of the spectral coefficients are real. With the assumption that the input signal is periodic, the DFT magnitude will be spatially invariant and it is worthy to be mentioned that it is not true for DCT. Moreover, while imposing periodicity into the time signal, the DCT does not bring discontinuities, but in the DFT case, the time signal is assumed to be periodic which results in discontinuities introduction in the time domain and of course there are resultant artifacts which are brought into the frequency domain. Though, since an even symmetry is supposed while truncating the time signal,
there will not be any discontinuities and any resultant artifact in the DCT.

5. SIMULATION

In this section, some simulation results are shown to demonstrate the gains of estimating PU signals directly from compressed measurements in CRNs. A radio spectrum is considered with a frequency range between 0Hz and 6MHz occupied by [0,5] users, as it is presented in Fig. 1. This figure illustrates the signal spectrum model with three different active primary signals positioned at 0.5 MHz – 1.5 MHz, 1.5 MHz – 2.5 MHz, 4.5 MHz – 5.5 MHz the primary signals which it is assumed to have random phases will be contaminated by using a zero-mean additive Gaussian white noise (AGWN). This makes the signal noise ratio (SNR) be 10dB. Later, the spectrum will be moved through the DCT. Duty of DCT is to transforms an array of real data points into its real spectrum. Threshold window is used to eradicate the small coefficients prior to the step that compressive sensing is applied to the DCT spectrum. The resulted spectrum will be multiplied by the measurement matrix.

![Fig 1](image-url)

Fig 1: (a) Representation the original spectrum of noiseless spectrum with three different active primary signals. (b) Discrete spectrum of real data created by DCT.

The output of compressive sensing would be the observation vector. This output is then directed to the AWGN channel with the aim of being transmitted. Later, at the receiver site the compressed DCT coefficient will be decompressed and reconstructed by employing an optimization technique like $l_1$. At last, the received spectrum is delivered through the IDCT to finally recover the spectrum. For showing distinction, reconstructed spectrums with different methods are given out in Fig. 2 with the same number of samples. Fig. 2(a) displays the result from standard LASSO based sparse spectrum recovery. Also the results from TS-CSS and WBSR based spectrum sensing schemes are illustrated in Fig. 2(b) and Fig. 2(c). The spectrum sensing enactment from the proposed scheme is described in Figure 2(d).
Fig 2: The compressed wideband spectrum estimation by reconstructed signal with using of ways the standard LASSO, the TS-CSS, the WBSR and the proposed ARMAND

In this paper, the computational complexity for the presented algorithm can be measured by using terms like the average CPU time which can be acquired from a total of 1000 trials for typical signals with lengths 2000. All simulations are performed on a laptop with an Intel(R) core(TM) i3 2.40 GHz processor and the CPU time was calculated by the Matlab (version 2010a). Table 1 illustrates the simulation outputs. From these outputs and results we can see that the compressed ratio (K/L) is about 50%. Note that the introduced ARMAND scheme is more efficient in comparison with others

In 1000 trials simulations, the average computing periods for the standard LASSO, the TS-CSS and the WBSR are 9.082 seconds, 2.853 seconds, 0.534 seconds on our hardware respectively. Nonetheless for presented approach, the average CPU time is 0.097s which is way faster and better that its competitors.

Table 1: Comparison numerical between proposed method by others methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Signal length</th>
<th>Compressed rate (K/L) (%)</th>
<th>Average CPU time(s)</th>
<th>Reconstruction Error 120 samples: $\frac{| \hat{X}_f - X_f |}{|X_f|}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LASSO</td>
<td>120</td>
<td>50</td>
<td>9.082</td>
<td>0.765</td>
</tr>
<tr>
<td>TS-CSS</td>
<td>120</td>
<td>50</td>
<td>2.853</td>
<td>0.349</td>
</tr>
<tr>
<td>WBSR</td>
<td>120</td>
<td>50</td>
<td>0.534</td>
<td>0.208</td>
</tr>
<tr>
<td>Proposed ARMAND</td>
<td>120</td>
<td>50</td>
<td>0.097</td>
<td>0.157</td>
</tr>
</tbody>
</table>
5. CONCLUSION

With the aim of sensing and transmission workload amount reduction in cognitive radio (CR) nodes, compressive sensing is applied for collaborative spectrum sensing in cognitive radio networks. In this paper, we proposed a method that is able to provide spectrum sensing by using measurement matrix and also is able to reconstruct the bands signals for CRNs without previously knowing the bands place information. This can expand the spectrum sensing efficiency and decrease the sensing time. Simulation results indicate that the presented ARMAND has a higher spectrum sensing sensitivity and precision and better process rapidity.

REFERENCES


