Vol. 5, No. 10 October 2014 ISSN 2079-8407
Journal of Emerging Trends in Computing and Information Sciences

http://www.cisjournal.org

Vague Li – Ideals on Lattice Implication Algebra

[1] T. Anitha, [2] V. AmarendraBabu
[1] Assistant Professor of Mathematics, K.L.University, Vaddeswaram, Guntur, A.P, India
[2] Assistant Professor, Department of Mathematics, Acharya Nagarjuna University, Nagarjuna Nagar –522 510
[1] anitha.t537@gmail.com, [2] amarendravelisela@ymail.com

ABSTRACT

We introduce the concept of vague LI – ideals on lattice implication algebra by linking the vague set and LI-ideal theory of Lattice implication algebras. The properties and equivalent characterizations of vague LI – ideals are investigated. We study the relationship between v- filters, vague LI - ideals and LI – ideals.

Keywords: *Lattice implication algebras, Li – ideals, Vague LI – ideals.*

1. INTRODUCTION

In order to research the logical system whose proportional value is given lattice, Y. XU [1] proposed the concept of lattice implication algebras, and discussed their some properties. Y.XU, Y.B. Jun and E.H. Roh [3] introduced the notion of LI – ideals of a lattice implication algebras, and discussed their some properties. Y.XU and Y.B. Jun [4] introduced the notion of fuzzy LI – ideals of a lattice implication algebras.

The concept of fuzzy set was introduced by Zadesh. Since then this idea has been applied to other algebraic structures such as groups, rings etc. With the development of fuzzy set, it is widely used in many fields. Meanwhile, the deficiency of fuzzy sets is also attracting attention. Such as fuzzy set is single function, it cannot express the evidence of supporting and opposing. Based on this reason, the concept of vague set [7] introduced by Gau in 1993. Ranjit Biswas [8] initiated the study of vague algebra by studying vague groups. Vague sets as a extension of fuzzy sets, the idea of vague sets is that the membership of every element can be divided into two aspects including supporting and opposing.

The object of this paper is to make a study of vague LI – ideals and its properties on lattice implication algebras L.

2. PRELIMINARIES

In this section we collect important results which were already proved for our use in the next section.

Definition 2.1:

[1] Let $(L, \vee, \wedge, {}', 0, I)$ be a complemented lattice with the universal bounds 0, I. $\rightarrow$ is another binary operation of L. $(L, \vee, \wedge, \rightarrow, {}', 0, I)$ is called a lattice implication algebra, if the following axioms hold, $\forall$ x, y, z ϵ L,

(I_1) $x \rightarrow (y \rightarrow z) = y \rightarrow (x \rightarrow z)$;
(I_2) $x \rightarrow x = I$;
(I_3) $x \rightarrow y = y' \rightarrow x'$;
(I_4) $x \rightarrow y = y \rightarrow x = I$ implies $x = y$;
(I_5) $(x \rightarrow y) \rightarrow y = (y \rightarrow y) \rightarrow x$
(L_1) $(x \vee y) \rightarrow z = (x \rightarrow z) \wedge (y \rightarrow z)$;
(L_2) $(x \wedge y) \rightarrow z = (x \rightarrow z) \vee (y \rightarrow z)$.

Definition 2.2:

[1] A lattice implication algebra $(L, \vee, \wedge, \rightarrow, {}', 0, I)$ is said to be a lattice H implication algebra if it satisfy the following axiom:

$$x \vee y \vee ((x \wedge y) \rightarrow z) = I, \forall\ x, y, z$$

Definition 2.3:

[3] Let A be a subset of a lattice implication algebra L. A is said to be an LI - ideal of L if it satisfies the following conditions:

(1) $0 \in A$;

(2) $\forall x, y \in L$, $(x \rightarrow y)' \in A$ and $y \in A$ implies $x \in A$.

Definition 2.4:

[5] Let A be a subset of a lattice implication algebra L. F is said to be a filter of L if it satisfies the following conditions:

(1) $I \in F$;

(2) $\forall x, y \in L$, $x \in F$ and $(x \rightarrow y) \in F$ implies $y \in F$.

Theorem 2.5:

[5] Let L be lattice implication algebra, then for any x, y, z ϵ L, the following conclusions hold:

(1) If $I \rightarrow x = I$ then $x = I$;
(2) $I \rightarrow x = x$ and $x \rightarrow 0 = x$;
(3) $0 \rightarrow x = I$ and $x \rightarrow I = I$;
(4) $x \leq y$ if and only if $x \rightarrow y = I$;
(5) $x \rightarrow y \geq x' \vee y$;
(6) $(x \rightarrow z) \rightarrow (x \rightarrow y) = ((z \wedge x) \rightarrow y)$
$= (z \rightarrow x) \rightarrow (z \rightarrow y)$.

Definition 2.6:

[7]. Let L_1 and L_2 is lattice implication algebras. A map f: $L_1 \rightarrow L_2$ is called an implication homomorphism if $f(x \rightarrow y) = f(x) \rightarrow f(y)$ for all x, y ϵ L_1.

Vol. 5, No. 10 October 2014 ISSN 2079-8407
Journal of Emerging Trends in Computing and Information Sciences

http://www.cisjournal.org

Moreover, if f satisfies the following conditions:

$f(x \wedge y) = f(x) \wedge f(y)$
$f(x \vee y) = f(x) \vee f(y)$
$f(x') = (f(x))'$

For all x, y $\epsilon\, L_1$, we say that f is a lattice implication homomorphism.

Definition 2.7:

[8] A vague set A in the universal of discourse X is characterized by two membership functions given by:

(1) A truth membership function $t_A : X \to [0,1]$ and
(2) A false membership function $f_A : X \to [0,1]$,

Where $t_A(x)$ is a lower bound of the grade of membership of x derived from the "evidence for x", and $f_A(x)$ is a lower bound on the negation of x derived from the "evidence against x" and $t_A(x) + f_A(x) \leq 1$.Thus the grade of membership of x in the vague set A is bounded by subinterval $[t_A(x), 1 - f_A(x)]$ of [0, 1]. The vague set A is written as $A = \{\langle x, [t_A(x), f_A(x)] \rangle / x \,\epsilon\, X\}$.

Where the interval $[t_A(x), 1 - f_A(x)]$ is called the value of x in the vague set A and denoted by $V_A(x)$.

Definition 2.8:

[8] A vague set A of a universe X with $t_A(x) = 0$ and $f_A(x) = 1$ for all x ϵ X, is called the zero vague set of X.

Definition 2.9:

[8] A vague set A of a universe X with $t_A(x) = 1$and $f_A(x) = 0$ for all x ϵ X, is called the unit vague set of X.

Definition 2.10:

[8] Let A be a vague set of a universe X with the truth membership function t_A and the false membership function f_A. . For any $\alpha,\beta \,\epsilon\, [0, 1]$ with $\alpha \leq \beta$, the (α,β) – cut or vague cut of a vague set A is a crisp subset $A_{(\alpha,\beta)}$ of the set X given by $A_{(\alpha,\beta)} = \{x \,\epsilon\, X / V_A(x) \geq [\alpha,\beta]\}$.

Definition 2.11:

[8] The α – cut, A_α of the vague set A is the (α, α) – cut of A and hence given by $A_\alpha = \{x \,\epsilon\, X / t_A(x) \geq \alpha\}$.

Notation [8]:

Let I [0, 1] denote the family of all closed subintervals of [0, 1]. If $I_1 = [a_1, b_1]$, $I_2 = [a_2, b_2]$ are two elements of I [0, 1], we call $I_1 \geq I_2$ if $a_1 \geq a_2$ and $b_1 \geq b_2$. We define the term IMAX to mean the maximum of two intervals as
$IMAX[I_1, I_2] = [\max\{a_1, a_2\}, \max\{b_1, b_2\}]$.

Similarly, we can define the term imin of any two intervals.

Definition 2.12:

[8] The intersection of two vague sets A and B with respective truth membership functions and the false membership functions t_A, t_B, f_A and $f_B$is a vague set C = A ∩ B, whose truth membership function and false membership functions are related to those of A and B by $t_C = \min\{t_A t_B\}$, $1 - f_C = \min\{1 - f_A, 1 - f_B\} = 1 - \max\{f_A, f_B\}$.

Definition 2.13:

[6] Let A be a vague set of lattice implication algebra L. A is said to be a v – filter of L if it satisfies the following conditions:

(1) $\forall x \epsilon L$, $V_A(I) \geq V_A(x)$,

(2) $\forall x, y \,\epsilon\, L$, $V_A(y) \geq \text{imin}\{V_A(x \to y), V_A(x)\}$.

3. VAGUE LI - IDEALS

Definition 3.1:

Let A be a vague set of a lattice implication algebra L. A is said to be a vague

LI - ideal of L if it satisfies the following conditions:

(1) $\forall x \epsilon L$, $V_A(0) \geq V_A(x)$,
(2) $\forall x, y \,\epsilon\, L$, $V_A(x) \geq \text{imin}\{V_A((x \to y)'), V_A(y)\}$.

By the definition 3.1 and the definition of V_A, the following theorem is obviously:

Theorem 3.2:

Vague set A of L is a vague LI – ideal of L, if and only if, for any $x,y \epsilon L$:

(1) $t_A(0) \geq t_A(x)$ and $1 - f_A(0) \geq 1 - f_A(x)$;
(2) $t_A(x) \geq \min\{t_A((x \to y)'), t_A(y)\}$ and $1 - f_A(x) \geq \min\{1 - f_A((x \to y)'), 1 - f_A(y)\}$.

Example 3.3:

Let L = {0, a, b, c, d, I} be a set with Cayley table as follows:

→	0	a	b	c	d	I
0	I	I	I	I	I	I
a	c	I	b	c	b	I
b	d	a	I	b	a	I
c	a	a	I	I	a	I
d	b	I	I	b	I	I
I	0	a	b	c	d	I

Vol. 5, No. 10 October 2014 ISSN 2079-8407
Journal of Emerging Trends in Computing and Information Sciences

http://www.cisjournal.org

Define $'$, $\vee$ *and* $\wedge$ operations on L as follows:
$x' = x \rightarrow 0$,
$x \vee y = (x \rightarrow y) \rightarrow y$,
$x \wedge y = ((x' \rightarrow y') \rightarrow y')'$ for all x, y ϵ *L*.

Then $(L, \vee, \wedge, \rightarrow, ', 0, I)$ is a lattice implication algebra [5]. Let A be a vague set of L defined by

$$A = \{\langle 0, [0.7, 0.2]\rangle, \langle a, [0.5, 0.3]\rangle, \langle b, [0.5, 0.3]\rangle, \langle c, [0.7, 0.2]\rangle, \langle d, [0.5, 0.2]\rangle, \langle I, [0.5, 0.3]\rangle, \langle I [0.5, 0.3]\rangle\}$$

Then A is a vague LI – ideal of L.

Corollary 3.4:

Zero vague set and Unit vague set are trivial vague LI – ideals of lattice implication algebra L.

Theorem 3.5:

Every vague LI- ideal A of a lattice implication algebra L is order reversing.

Proof:

Let A be a vague LI – ideal of L.If x, y ϵ L and $x \leq y$, then$(x \rightarrow y)' = I' = 0$, and so $t_A(x) \geq \min\{t_A((x \rightarrow y)'), t_A(y)\} = \min\{t_A(0), t_A(y)\} = t_A(y)$ and $1- f_A(x) \geq \min\{1 - f_A((x \rightarrow y)'), 1 - f_A(y)\}, = \min\{1 - f_A(0), 1 - f_A(y)\} = 1 - f_A(y)$. So $V_A(x) = [t_A(x), 1- f_A(x)] \geq [t_A(y), 1- f_A(y)] = V_A(y)$. This shows that A is order reversing.

Definition 3.6:

Let A be a vague set of a lattice implication algebra L. A is said to be a vague lattice ideal of L if it satisfies the following conditions:

(1) $y \leq x$ then $V_A(y) \geq V_A(x)$,
(2) $V_A(x \vee y) \geq \text{imin}\{V_A(x), V_A(y)\}$ for x, y ϵ L.

Example 3.7:

Let $(L, \vee, \wedge, \rightarrow, ', 0, I)$ is a lattice implication algebra in example 3.3 . Define a vague set A of L by

$$A=\{\langle 0, [0.7, 0.2]\rangle. \langle a, [0.5, 0.3]\rangle, \langle b, [0.5, 0.3]\rangle, \langle c, [0.5, 0.3]\rangle, \langle d, [0.7, 0.2]\rangle, \langle I, [0.5, 0.3]\rangle\}.$$

Then A is a vague lattice ideal of L.

Theorem 3.8:

Every vague LI – ideal of a lattice implication algebra L is a vague lattice ideal of L.

Proof:

Let A be a vague LI – ideal of a lattice implication algebra L.

Theorem 3.5 shows that $V_A(y) \geq V_A(x)$ if $y \leq x$.
By $((x \vee y) \rightarrow y)' = ((x \rightarrow y) \wedge (y \rightarrow y))'$ (by L_1)
$= (x \rightarrow y)'$
$\leq (x' \vee y)'$ (by theorem 2.5)
$= (x')' \wedge y'$
$= x \wedge y'$
$\leq x$,

We get $V_A(((x \vee y) \rightarrow y)') \geq V_A(x)$.
From the definition of vague LI – ideal , we have
$V_A(x \vee y) \geq \text{imin}\{V_A(((x \vee y) \rightarrow y)'), V_A(y)\}$
$\geq \text{imin}\{V_A(x), V_A(y)\}$.
Hence every vague LI – ideal of L is a lattice ideal of L.

Remark 3.9:

Converse of the above theorem need not to be true. For example the vague set A defined in example 3.7 is a vague lattice ideal of L, but not a vague LI- ideal for $V_A(a) \ngeq \text{imin}\{V_A((a \rightarrow d)'), V_A(d)\}$.

Theorem 3.10:

In a lattice H implication algebra L, every vague lattice ideal is a vague LI – ideal.

Proof:

Let A be a vague lattice ideal of a lattice H implication algebra L.Since $0 \leq x$, it follows that $V_A(0) \geq V_A(x)$ for any x ϵ L.Since L is a lattice H implication algebra, we have $y \vee (x \rightarrow y)' = y' \rightarrow (x \rightarrow y)' = (x \rightarrow y) \rightarrow y = x \vee y \geq x$,We have $V_A(x) \geq V_A(y \vee (x \rightarrow y)') \geq \text{imin}\{V_A(y), V_A((x \rightarrow y)')\}$.Hence A is a vague LI – ideal of L.

Example 3.11:

Let L = {0, a, b, I} be a set with Cayley table as follows:

$\rightarrow$	0	a	b	I
0	I	I	I	I
a	b	I	b	I
b	a	a	I	I
I	0	a	b	I

Define $'$, $\vee$ *and* $\wedge$ operations on L as follows:
$x' = x \rightarrow 0$,
$x \vee y = (x \rightarrow y) \rightarrow y$,
$x \wedge y = ((x' \rightarrow y') \rightarrow y')'$ for all x, y ϵ *L*.
Then $(L, \vee, \wedge, \rightarrow, ', 0, I)$ is a lattice H implication algebra[5]. Define a vague set A of L by

$$A=\{\langle 0, [0.7, 0.2]\rangle, \langle a, [0.5, 0.3]\rangle, \langle b, [0.5, 0.3]\rangle, \langle I, [0.5, 0.3]\rangle\}$$

Then A is both vague LI – ideal and vague lattice ideal of L.

Theorem 3.12:

Let A be a vague set of a lattice implication algebra L. Then A is a vague LI – ideal if and only if $A_{(\alpha,\beta)}$ is an LI – ideal when $A_{(\alpha,\beta)} \neq \emptyset$, $\alpha,\beta \ \epsilon$ [0, 1].

Proof:

Let A be a vague LI – ideal of L and $\alpha, \beta \ \epsilon$ [0, 1] such that $A_{(\alpha,\beta)} \neq \emptyset$. Clearly 0 ϵ $A_{(\alpha,\beta)}$. Suppose x, y ϵ L,(x

Vol. 5, No. 10 October 2014 ISSN 2079-8407
Journal of Emerging Trends in Computing and Information Sciences

http://www.cisjournal.org

$\to y)' \in A_{(\alpha,\beta)}$ and $y \in A_{(\alpha,\beta)}$.Then $V_A((x \to y)') \geq [\alpha,\beta]$ and $V_A(y) \geq [\alpha,\beta]$. It follows that $V_A(x) \geq$ imin{ $V_A((x \to y)')$, $V_A(y)$} $\geq[\alpha,\beta]$,So that $x \in A_{(\alpha,\beta)}$. Hence $A_{(\alpha,\beta)}$is an LI – ideal of L.

Conversely, suppose that $A_{(\alpha,\beta)} \neq \emptyset$is an LI – ideal of L where $\alpha,\beta \in [0, 1]$. For any $x \in L$, $x \in A_{A(x)}$, it follows that $A_{A(x)}$ is an LI – ideal of L and hence $0 \in A_{A(x)}$, that is $V_A(0) \geq V_A(x)$.

For any $x, y \in L$, let $[\alpha,\beta] = \min\{V_A((x \to y)')$, $V_A(y)\}$, it follows that $A_{(\alpha,\beta)}$is an LI – ideal and $(x \to y)' \in A_{(\alpha,\beta)}$, $y \in A_{(\alpha,\beta)}$, this implies that $x \in A_{(\alpha,\beta)}$.So $V_A(x) \geq [\alpha, \beta]$ =imin $\{V_A((x \to y)'), V_A(y)\}$.Hence A is vague LI – ideal of L.

Corollary 3.13:

Let A be a vague set of a lattice implication algebra L. Then A is a vague LI - ideal of L if and only if A_α is an LI – ideal of L when $A_\alpha \neq \emptyset$, $\alpha \in [0, 1]$.

Difinition3.14:

Let A be a vague set of a lattice implication algebra L. We define a new set A′ of L as $A'(x) = A(x')$ for $x \in L$.

Theorem3.15:

Let A be a vague set of a lattice implication algebra L. Then A is a v- filter of L if and only if A′ is a vague Li – ideal of L.

Proof:

Let A is a v- filter of L and consider $A_\alpha \neq$ □, where $\alpha \in [0, 1]$is a α- cut set of A. For any $x \in L$, $x \in A_\alpha$, we have $V_A(x) \geq (\alpha, \alpha)$By A is a filter, we have $V_A(I) \geq V_A(x) \geq (\alpha, \alpha)$, therefore $I \in A_\alpha$. Let $x, y \in L$ and $x \in A_\alpha$,$(x \to y) \in A_\alpha$, then $V_A(x) \geq [\alpha,\alpha]$, $V_A(x \to y) \geq [\alpha,\alpha]$. It follows that $V_A(y) \geq$ imin$\{V_A(x \to y)$, $V_A(x)\} \geq [\alpha,\alpha]$,therefore $y \in A_\alpha$ .Then A_α is an filter of L. It is obvious that if A is a vague set of L then $(A')_\alpha = (A_\alpha)'$, $\forall \alpha \in [0, 1]$.By the theorem 2.7 in [3], we have $(A_\alpha)'$ is an LI- ideal of L.By the corollary 3.13, we have A′ is a vague LI – ideal of L. Hence the theorem.

Theorem 3.16:

Any LI – ideal I of a lattice implication algebra L is a vague cut LI – ideal of some vague Li – ideal of L.

Proof:.

Consider a vague set A of L, $V_A(x) = [\alpha, \alpha]$ if $x \in I$ and $V_A(x) = [0, 0]$ if $x \notin I$, where $\alpha \in [0, 1]$.Since I is an LI – ideal of L, we have $0 \in I$. Therefore $V_A(0) = [\alpha, \alpha] \geq V_A(x)$.For any $x, y \in L$, if $x \in I$ then $V_A(x) = [\alpha, \alpha] \geq$ imin$\{V_A((x \to y)'), V_A(y)\}$.

Suppose $x \notin I$, then $y \notin I$ or $(x \to y)' \notin I$, and so $V_A(x) = [0, 0] =$ imin$\{V_A((x \to y)')$, $V_A(y)\}$. Therefore A is a vague LI – ideal of L.

Theorem3.17:

Let A be a vague LI – ideal of a lattice implication algebra. Then $I = \{x \in L / V_A(x) = V_A(0)\}$ is a LI – ideal of L.

Proof:

Since $I = \{x \in L / V_A(x) = V_A(0)\}$, obviously $0 \in I$.

Let $x, y \in L$ and $(x \to y)' \in I$, $y \in I$ then $V_A((x \to y)' = V_A(y) = V_A(0)$. A is a vague LI – ideal, then we have $V_A(x) \geq$ imin$\{V_A((x \to y)')$, $V_A(y)\} = V_A(0)$.
And $V_A(0) \geq V_A(x)$, then $V_A(x) = V_A(0)$ .Thus $x \in I$. It follows that I is a LI – ideal of L.

Example 3.18:

In example 3.3

$$A=\{\langle 0,[0.7,0.2]\rangle,\langle a,[0.5,0.3]\rangle,\langle b,[0.5,0.3]\rangle,$$
$$\langle c,[0.7,0.2]\rangle,\langle d,[0.5,0.3]\rangle,\langle I,[0.5,0.3]\rangle\}$$

is a vague LI – ideal of L. Define $I = \{x \in L / V_A(x) = V_A(0)\}$, then $I = \{0, c\}$ and clearly $I = \{0, c\}$ is an LI- ideal of L.

Theorem 3.19:

Let L_1 and L_2 be two lattice implication algebras, the mapping f: $L_1 \to L_2$ is a lattice implication homomorphism.

(1) If B is a vague LI – ideal of L_2 then $f^{I}(B)$ is a vague LI – ideal of L_1.
(2) f A is a vague LI – ideal of L_1 and f is bijection, then f (A) is a vague LI – ideal of L_2.

Proof:

(1) For any $x, y \in L_1$, By f (0) = 0,itfollows that
$f^{I}(V_B)(x) = V_B(f(x)) \leq V_B(0) = V_B(f(0)) = f^{I}(V_B)(0)$.

$$\begin{aligned} f^{I}(V_B)(x) &= V_B(f(x)) \\ &\geq \text{imin}\{ V_B((f(x) \to f(y))')\ V_B f(y) \} \\ &= \text{imin}\{ V_B((f(x \to y)')\ ,\ V_B f(y) \} \\ &= \text{imin}\ \{f^{I}(V_B)((x \to y)'),\ f^{I}(V_B)(y)\}. \end{aligned}$$

Therefore $f^{I}(B)$ is a vague LI – ideal of L_1.
(2).For any $x, y \in L_2$, there exist $u, v \in L_1$ such that f(u) = x and f(v) = y, it follows that

$$\begin{aligned} f(V_A)(x) &= V_A(f^{I}(x)) = V_A(u) \leq V_A(0) = V_A(f^{I}(0)) \\ &= f(V_A)(0). \\ f(V_A)(x) &= V_A(u) \geq \text{imin}\{V_A((u \to v)'),\ V_A(v)\} \\ &= \text{imin}\{f(V_A)((x \to y)'),\ f(V_A)(y)\}. \end{aligned}$$

Hence f (A) is a vague LI – ideal of L_2.

Theorem 3.20:

Let A be a vague set of a lattice implication algebra L. Then A is a vague LI – ideal of L if and only if for any $x, y \in L$, A satisfies

(1) $x \leq y$ then $V_A(x) \geq V_A(y)$
(2)$V_A(x \oplus y) \geq$ imin $\{V_A(x)$, $V_A(y)\}$

Vol. 5, No. 10 October 2014 ISSN 2079-8407
Journal of Emerging Trends in Computing and Information Sciences

http://www.cisjournal.org

Proof:

Let A be a vague LI – ideal of L. Clearly (1) holds. Since $x \geq ((x \oplus y) \rightarrow y)'$, we have $V_A(x) \leq V_A((x \oplus y) \rightarrow y)')$. It follows that $V_A(x \oplus y) \geq \text{imin}\{V_A((x \oplus y) \rightarrow y)'), V_A(y)\}$ $\geq \text{imin}\{V_A(x), V_A(y)\}$

Conversely, suppose that the vague set A of L satisfies the above two conditions. Since $0 \leq x$ for all $x \in L$, we have $V_A(0) \geq V_A(x)$.

By $x \leq ((x \rightarrow y)' \oplus y)$, we get $V_A(x) \geq V_A((x \rightarrow y)' \oplus y) \geq \text{imin}\{V_A(x \rightarrow y)', V_A(y)\}$ Hence A is vague LI – ideal of L.

Corollary 3.21:

A vague set A of a lattice implication algebra L is a vague LI – ideal of L if and only if for any $x, y, z \in L$; if $x \rightarrow (y' \rightarrow z) = I$ then $V_A(x) \geq \text{imin}\{V_A(z), V_A(y)\}$.

Theorem 3.22:

Let A and B be too vague LI – ideals of L, then $A \cap B$ is also vague LI – ideal of L.

Proof:

Let $x, y, z \in L$ such that $x \rightarrow (y' \rightarrow z) = I$

Since A and B are two vague LI – ideals of L, we have $V_A(x) \geq \text{imin}\{V_A(z), V_A(y)\}andV_B(x) \geq \text{imin}\{V_B(z), V_B(y)\}$.That is, $t_A(x) \geq \min\{t_A(z), t_A(y)\}$ and $1 - f_A(x) \geq \min\{1- f_A(z), 1- f_A(y)\}andt_B(x) \geq \min\{t_B(z), t_B(y)\}$ and $1- f_B(x) \geq \min\{1- f_B(z), 1 - f_B(y)\}$.

Since $t_{A \cap B}(x) = \min\{t_A(x), t_B(x)\}$
$\geq \min\{\min\{t_A(z), t_A(y)\}, \min\{t_B(z), t_B(y)\}\}$
$= \min\{\min\{t_A(z), t_B(z)\}, \min\{t_A(y), t_B(y)\}\}$
$= \min\{t_{A \cap B}(z)), t_{A \cap B}(y)\}$

Similarly, we can prove that
$1 - f_{A \cap B}(x) \geq \min\{1 - f_{A \cap B}(z), 1 - f_{A \cap B}(y)\}$.

Therefore, $V_{A \cap B}(x) = [t_{A \cap B}(x), 1 - f_{A \cap B}(x)]$
$\geq \text{imin}\{V_{A \cap B}(z), V_{A \cap B}(y)\}$.

Hence $A \cap B$ is a vague LI – ideal of L.

Theorem 3.23:

Let A be a vague set of a lattice implication algebra L. Then A is a vague LI – ideal of L if and only if for any $x, y, z \in L$, A satisfies

(1) $V_A(0) \geq V_A(x)$,
(2) $V_A(x \rightarrow z) \geq \text{imin}\{V_A((x \rightarrow z) \rightarrow y)'), V_A(y)\}$

Proof:

Let A be vague LI – ideal of L, obviously A satisfies the above two conditions.

Conversely, suppose that A is a vague set of L and satisfies the above conditions. Taking $x = I$ in (ii), we have $V_A(z) \geq \text{imin}\{V_A(z \rightarrow y)', V_A(y)\}$ Hence A is vague LI – ideal of L.

Theorem 3.24:

Let A be a vague set of a lattice implication algebra L. Then A is a vague LI – ideal of L if and only if for any $x, y, z \in L$, A satisfies

(i) $V_A(0) \geq V_A(x)$,
(ii) $V_A(x \rightarrow z) \geq \text{imin}\{V_A((z \rightarrow y)')), V_A(x \rightarrow y)\}$

Proof:

Let A be vague LI – ideal of L, obviously A satisfy the first condition We have.
$V_A(x \rightarrow z) \geq \text{imin}\{V(((x \rightarrow z) \rightarrow (x \rightarrow y))'), V_A(x \rightarrow y)\}$,
Consider $(z \rightarrow y) \rightarrow ((x \rightarrow z) \rightarrow (x \rightarrow y))$
$= (z \rightarrow y) \rightarrow ((x \wedge z) \rightarrow y))$ (by theorem 2.5)
$= (z \rightarrow y) \rightarrow ((z \rightarrow x) \rightarrow (z \rightarrow y))$
$= (z \rightarrow x) \rightarrow ((z \rightarrow y) \rightarrow (z \rightarrow y))$
$= (z \rightarrow x) \rightarrow I$
$= I$
So $(z \rightarrow y) \leq ((x \rightarrow z) \rightarrow (x \rightarrow y))$.
we have $V_A((z \rightarrow y)') \leq V_A(((x \rightarrow z) \rightarrow (x \rightarrow y))')$
It follows that

$V_A(x \rightarrow z) \geq \text{imin}\{V_A((z \rightarrow y)'), V_A(x \rightarrow y)\}$

Conversely, suppose that the vague set A of L and satisfies the above two conditions. Taking $x = I$ in (ii), then we have $V_A(z) \geq \text{imin}\{V_A((z \rightarrow y)'), V_A(y)\}$ Hence A is a Vague LI – ideal of L.

Theorem 3.25:

Let A be a vague set of a lattice implication algebra L. Then A is a vague LI – ideal of L if and only if for any $x, y, z \in L$, A satisfies

(1)$V_A(0) \geq V_A(x)$,
(2)$V_A(z) \geq \text{imin}\{V_A(x), V_A(y), V_A(((z \rightarrow x)' \rightarrow y)')\}$

Proof:

Let A be vague LI – ideal of L, obviously A satisfy the first condition. For any $x, y, z \in L$, we have $V_A(z) \geq \text{imin}\{V_A((z \rightarrow x)'), V_A(x)\}$and
$V_A((z \rightarrow x)') \geq \text{imin}\{V_A(((z \rightarrow x)' \rightarrow y)', V_A(y))\}$
Hence

$V_A(z) \geq \text{imin}\{V_A(x), V_A(y), V_A(((z \rightarrow x)' \rightarrow y)')$

Conversely, suppose that the vague set A of L and satisfies the above two conditions. Taking $x = y$ in (ii), then we have $V_A(z) \geq \text{imin}\{V_A(x), V_A(((z \rightarrow x)' \rightarrow x)')\}$

Since $(z \rightarrow x)' \geq ((z \rightarrow x)' \rightarrow x)'$,
we have $V_A((z \rightarrow x)') \leq V_A((z \rightarrow x)' \rightarrow y)'\}$
It follows that $V_A(z) \geq \text{imin}\{V_A((z \rightarrow x)'), V_A(x)\}$
Hence A is a vague LI – ideal of L

Vol. 5, No. 10 October 2014 ISSN 2079-8407
Journal of Emerging Trends in Computing and Information Sciences

http://www.cisjournal.org

REFERENCES

[1] Y. Xu, Lattice implication algebras, J. Southwest Jiaotong University 28(1) (1993), 20 – 27.

[2] Y. Xu, Homomorphisms in lattice implication algebras, Proc. of 5th Many – valued Logical Congress of China (1992), 206 – 211.

[3] Y.Xu, Y.B. Jun and E.H. Roh, LI – ideals in lattice implication algebras, Bull. Korean Math. Soc. 35(1) (1998), 13 – 23.

[4] Y.Xu and Y.B. Jun, Fuzzy LI – ideals in lattice implication algebras, J. Fuzzy Math. 7(4) (1999), 997 - 1003.

[5] Y. Xu, D. Ruan, K.Y. Qin, Lattice – Valued Logic – an alternative approach to treat fuzziness and incomparability, Springer.

[6] Ya Qin and Yi Liu, V- filter on lattice implication algebras, Journal of Emerging Trends in Computing and Information Sciences, 3(9)(2012), 1298 – 1301.

[7] L. A. Zadeh, Fuzzy sets Information and Control 8 (1965), 338 – 353.

[8] Gai W. L., Buehrer D. J., Vague sets, IEEE Transactions on Systems, Man and Cybernetics, 23(20)(1993), 610 – 614.

[9] Eswarlal .T, vague ideals and normal vague ideals in semi rings, International journal of computational cognition, 6 (3), (2008), 60 – 65.

[10] Biswas R, Vague Groups, Int. J. Comput. Cngition, 4 (2) (2006), 20 – 23.