Model and Algorithm for Solving School Bus Problem

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Abstract

School bus routing problem has been a significant concern of most people related to school and school bus system as one of vehicle routing problems. Making an appropriate problem formulation depends on how to reflect the realities of the problem. And, as the problem scope becomes wider, the problem can’t be solved only with the exact methods. So, there is need to develop an efficient heuristic method to solve more complicated problem. In this study, the model for school bus routing problem is proposed, and a heuristic algorithm for solving the proposed model is suggested. The model is formulated as a mixed-integer programming problem. To validate the model, several random small network problems are solved by using the commercial optimization package CPLEX. Also, a heuristic algorithm based on harmony search is proposed to solve this problem. The results of the heuristic are compared with the results obtained from exact solution by CPLEX to validate and evaluate the heuristic algorithm. Computation results show that the solution by the heuristic was exactly the same as that of exact method using CPLEX. But, the heuristic produces the same results in a very short time.

Keywords: School bus problem, heuristic, harmony search

1. INTRODUCTION

SBRP (School bus routing problem) has been a significant concern of experts and researchers related to school and school bus system as one of VRP (vehicle routing problems). SBRP is to efficiently transport students from their origins to a school using a given number of buses. As a school bus service area increases, students and the number of buses that are necessary for service increase, and it is hard to solve SBRP and get the optimal solution within the desired time by exact method. Therefore, the heuristic method is needed to solve complex problems efficiently. HS (Harmony Search) as one of the optimization algorithms developed by Geem et al. (2001) [1] has been widely used in many areas such as music composition, project scheduling, university timetabling, internet routing, truss structure design, water network design, medical imaging and astronomical data analysis. Also, the excellence of HS has been proven through many practices. However, there are few practices of HS applied in the field of transportation.

Therefore, in this study, SBPR model is proposed and a heuristic algorithm is presented for solving the proposed model. For this work, first, a model of SBRP is formulated as a mixed-integer programming problem. To validate the model, several random network problems are solved by exact method using the commercial optimization package CPLEX. Also, the same problems are solved by the proposed heuristic algorithm using harmony search and the results of the proposed heuristic are compared with the results obtained from exact method.

2. LITERATURE REVIEW

The surveys on models and algorithm developed for DBRP are described by Desrosiers et al. (1981) [2] and Park and Kim (2010) [3]. According to Desrosiers et al. (1981), SBRP can be divided into 5 steps: data preparation, bus stop selection, bus route generation, school bell time adjustment and route scheduling. This study belongs to bus route generation and route scheduling steps among 5 steps. Bennett and Gazis (1972) used saving algorithm to minimize total student-distance [4]. Bodin and Berman (1979) built routes using route-first cluster-second and then improved the routes using 3-opt algorithm [5]. Tsay and Fricker (1988) formulated the model as multi-objective function and solved the model through three steps of processing [6]. Bowerman et al. (1995) applied cluster-first route-second to SBRP instead of route-first cluster-second used by Bodin and Berman (1979) [7]. Braca et al. (1997) solved SBRP of New York city for general and special students to minimize the number of buses needed for service. Geem et al. (2005) applied HS to a test network randomly generated and compared the result by HS with that by GA [9]. Consequentially, it showed that HS can get better solution than GA within shorter time.

Recently, meta-heuristics such as Simulated Annealing (SA), Tabu Search (TS), Genetic Algorithm (GA), Ant Colony Optimization (ACO) have been developed and applied in various combinatorial optimization problems. However, there are not many practices of meta-heuristics applied in SBRP, and it is expected that many research on SBRP will be done using these meta-heuristics in the near future.

2.1 School Bus Routing Problem

Many optimization problems in various fields have been solved using optimization techniques, such as linear programming (LP), non-linear programming (NLP), and dynamic programming (DP). However, their drawbacks generate demand for other types of algorithms, such as heuristic optimization approaches (simulated annealing, tabu search, and genetic algorithm). However, there are still some possibilities of devising new heuristic algorithms based on analogies with natural or artificial phenomena. Geem et al. (2001) developed a new heuristic algorithm mimicking the improvisation of music players, named HS.
The basic concept, steps and structures, and parameters of HS can be referred to Geem et al. (2001) [1].

2.1.1 The Basic Concept

HS was originated from an artificial phenomenon seek the better harmony on Jazz performance. Musical performances seek a best state (fantastic harmony) determined by aesthetic estimation, as the optimization algorithms seek a best state (global optimum) determined by objective function evaluation. Aesthetic estimation is determined by the set of the sounds played by joined instruments, just as objective function evaluation is determined by the set of the values produced by component variables; the sounds for better aesthetic estimation can be improved through practice after practice, just as the values for better objective function evaluation can be improved iteration by iteration. A brief presentation of these observations is shown Table 1.

Table 1: Comparison between optimization and musical performance (Geem et al., 2001)

<table>
<thead>
<tr>
<th>Comparison Factor</th>
<th>Optimization Process</th>
<th>Performance Process</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best state</td>
<td>Global Optimum</td>
<td>Fantastic Harmony</td>
</tr>
<tr>
<td>Estimated by</td>
<td>Objective Function</td>
<td>Aesthetic Standard</td>
</tr>
<tr>
<td>Estimated with</td>
<td>Values of Variables</td>
<td>Pitches of Instruments</td>
</tr>
<tr>
<td>Process unit</td>
<td>Each Iteration</td>
<td>Each Practice</td>
</tr>
</tbody>
</table>

2.1.2 Steps and Structures

The steps in the procedure of Harmony Search are as follows:

Step 1: Initialize a Harmony Memory (HM)
Step 2: Improve a new harmony
Step 3: If the new harmony is better than the worst harmony in HM, include the new harmony in HM
Step 4: If stopping criteria are not satisfied, go to step 2.

2.1.3 Parameters

Of course, the above assumes that all the parts of the global solution exist initially in HM. When this is not the case, in order to find global optimum, Harmony Search initiates a parameter, Harmony Memory Considering Rate (HMCR), which ranges from 0 to 1. If a uniformly generated value between 0 and 1 occurs above the current value of the HMCR, then HS finds notes randomly within the possible playable range without considering HM. A HMCR of 0.95 means that at the next step, the algorithm chooses a variable value from HM with a 95% probability.

For improving solutions and escaping local optima, yet another option may be introduced. This option mimics the pitch adjustment of each instrument for tuning the ensemble. For computation, the pitch adjustment mechanism is devised as shifting to neighboring values within a range of possible values. A Pitch Adjustment Rate (PAR) of 0.10 means that the algorithm chooses a neighboring value with 10% probability (an upper value with 5% or a lower value with 5%).

3. MODEL FORMULATION FOR SCHOOL BUS PROBLEM

3.1 Assumptions

In this study, single depot and single school are considered. A school has an arrival time window which means the allowable arrival time. Therefore, a school bus must arrive at school within the time window, that is, the bus must not arrive at school earlier than the earliest arrival time and later than the latest arrival time. It is assumed that the width of time window is 15 minutes. For example, if the arrival time window at school is (8, 8:15), the bus must arrive between 8 and 8:15. In the typical school bus routing problem, the locations of bus stops are determined by the way minimizing the average walking distance between students’ homes and the nearest bus stop. Therefore, we assume that the locations of bus stops are the same as those of demand points and in our problem, bus stop locations are considered to be given data. In addition, the available vehicles are considered to be homogenous type, which means that all vehicles have the same capacity.

3.2 Problem formulation

This section provides a mathematical formulation for school bus problem as a mixed-integer programming problem. The decision variables, constants, data sets used in this model formulation are defined follows.

3.2.1 Decision variables

\[ x_{ij}^k = \begin{cases} 1 & \text{If vehicle } k \text{ travels link } ij \\ 0 & \text{Otherwise} \end{cases} \hspace{1cm} k \in V, \ i \in DS, \ j \in SE \]

\[ t_i^k = \text{actual arrival time of vehicle } k \text{ at node } i, \ i \in TD, \ k \in V \]

3.2.2 Constants

\[ F = \text{fixed cost per school bus} \ ($/bus) \]
\[ R = \text{routing cost per unit travel time} \ ($/min) \]
\[ t_{ij} = \text{travel time between node } i \text{ and node } j \text{ (min),} \]
\[ i \in DS, \ j \in SE \]
\[ P_i = \text{demand (students) of node } i, \ i \in S \]
\[ Q_k = \text{capacity of vehicle } k, \ k \in V \]
\[ b = \text{average boarding time per } 1 \text{ student} \]
\[ T_e = \text{earliest arrival time at school} \]
\[ T_l = \text{latest arrival time at school} \]
\[ T = \text{maximum in-vehicle time} \]
3.2.3 Data Sets

- \( S \) = demand nodes (bus stops)
- \( D \) = a vehicle’s starting node, i.e. depot
- \( E \) = a vehicle’s ending node, i.e. school
- \( DS \) = all nodes which permit on out-flow, i.e. depot and bus stops, \( D \cup S \)
- \( SE \) = all nodes which permit on in-flow, i.e. school and bus stops, \( E \cup S \)
- \( TD \) = the set of all nodes, \( D \cup S \cup E \)
- \( V \) = the vehicle set

The proposed mathematical formulation is as follows:

Minimize

\[
Z = F \sum_{i \in D} \sum_{j \in S} x_{ij}^s + R \sum_{i \in DS} \sum_{j \in SE} x_{ij}^s t_{ij}
\]

subject to

\[
\sum_{k \in V} \sum_{j \in DS} x_{ij}^s = 1 \quad j \in S
\]

\[
\sum_{k \in V} \sum_{j \in SE} x_{ij}^s = 1 \quad i \in S
\]

\[
\sum_{k \in SE} x_{ij}^s - \sum_{j \in SE} x_{ij}^s = 0 \quad k \in V, \quad p \in S
\]

\[
\sum_{i \in DS} \sum_{j \in SE} x_{ij}^s - \sum_{i \in S} \sum_{j \in SE} x_{ij}^s = 0
\]

\[
\sum_{i \in DS} \left( \sum_{j \in SE} X_{ij}^k \right) \leq Q_k \quad k \in V
\]

\[
t_i^e + t_{ij} + p_i \times b \leq t_j^e + M(1 - X_{ij}^k) \quad k \in V, \ i \in DS, \ j \in SE
\]

\[
t_i^e + t_{ij} + p_i \times b \geq t_j^e + M( X_{ij}^k - 1) \quad k \in V, \ i \in DS, \ j \in SE
\]

\[
T_e \leq t_{ij}^e \leq T_f
\]

\[
t_i^e + p_i \times b \geq t_{ij}^e - T \quad i \in S, \ k \in V
\]

\[
\sum_{k \in V} \sum_{j \in S} x_{ij}^s \leq I \quad i \in D
\]

\[
\sum_{k \in V} \sum_{j \in E} x_{ij}^s \leq I \quad j \in E
\]

3.2.4 Objective Function

The objective of this problem is to minimize the total cost incurred in serving all students. That is, the total cost consists of the fixed costs of the number of vehicles and the routing costs.

3.2.5 Constraints

In addition, there are many constraints for this problem. Each demand node should be served by one vehicle (2-3). If a vehicle enters a node, it must exit that node (4). The number of vehicle leaving depot must equal the number of vehicle entering the school (5). There is a limit for the number of students on the bus as capacity constraint (6). Arrival time window at school, which means that school bus arrive at school between \( T_e \) and \( T_f \) (7-9). Earliest departure time at demand node \( i \) (bus stops), by which school bus cannot leave each bus stop before the earliest departure time (10). Each vehicle can leave one depot and arrive at one school only once, which means that the number of routes should be less than or equal to the number of buses (11-12).

4. EXPERIMENTAL RESULTS AND COMPARISON

4.1 Test Networks

To validate the proposed model in this study, several random network problems were made as shown in Table 2. There are 5 test networks and it is assumed that the fixed cost of a bus is $100,000/bus and the traveling cost is $105/minute.

Table 2: Test networks for experiments

<table>
<thead>
<tr>
<th>NO.</th>
<th>The number of nodes(stops)</th>
<th>The number of available buses</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6(4)</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>6(4)</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>9(7)</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>12(10)</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>20(18)</td>
<td>6</td>
</tr>
</tbody>
</table>

4.2 Solving problem by an exact method (Branch & Bound)

The process of optimizing routes and scheduling for SBRP by exact method is divided into two steps. First, it is necessary to prepare the program which can generate and code correctly the objective function and its constraints for each test network as the format of CPLEX Input.

In this study, C++ programming language was used to generate a CPLEX input file. Next, the output of this program is put into the CPLEX and a solution is obtained through optimal process in CPLEX.

4.3 Solving problem by a heuristic (Harmony Search)

The HS is applied to a SBRP as follows, Fig 2

The size of Harmony memory is 10 and HMCR is 0.9.
Fig 1: Solution steps of the proposed model using HS

4.4 Comparison of results

In this section, the results of a heuristic using HS are compared with the results of exact method (Branch-and-Bound) using CPLEX.

The HS was coded in Quick Basic and tested on 2.0 GHz Intel core 2 Duo CPU having 3.0GB RAM. The computational results are like Table 3. In case of HS, the values for the first network to the fourth network in table are results of 200 iterations and those for the fifth network are results of 1200 iterations.

Table 3: Computational results by exact method and HS

<table>
<thead>
<tr>
<th># nodes (stops)</th>
<th># available bus</th>
<th>Exact method</th>
<th>Harmony Search</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cal. Time(s)</td>
<td>O.F.</td>
<td># routes</td>
</tr>
<tr>
<td>6(4)</td>
<td>2</td>
<td>0.16</td>
<td>210,500</td>
</tr>
<tr>
<td>6(4)</td>
<td>3</td>
<td>0.78</td>
<td>210,500</td>
</tr>
<tr>
<td>9(7)</td>
<td>3</td>
<td>1343</td>
<td>306,195</td>
</tr>
<tr>
<td>12(10)</td>
<td>4</td>
<td>7926</td>
<td>307,980</td>
</tr>
<tr>
<td>20(18)</td>
<td>6</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

5. CONCLUSION

In this study, the model for school bus routing problem is proposed, and a heuristic algorithm for solving the proposed model is suggested. The model is formulated as a mixed-integer programming problem. To validate the model, several random small network problems are solved by exact method using the commercial optimization package CPLEX. Also, a heuristic algorithm based on harmony search is proposed to solve this problem. The results of the heuristic are compared with the results obtained from exact solution by CPLEX to validate and evaluate the heuristic algorithm.

As a result, the solution (objective function) by HS was exactly the same as that of exact method. But, HS produces the same results in a very short time. As the number of nodes exceeds 9, the computational time by exact method exponentially increases and it is impossible to get a solution within an appropriate time. But, HS found the identical feasible solution only after 1200 iterations and generated alternative solutions. It shows that a heuristic using HS can find a good solution of SBRP with a short time. However, it is not guaranteed that the solution of HS is the global optimal as the size of network increases.

Therefore, it is necessary to develop a method that can find good lower bound of objective function in a mathematical model. In this study, single depot and single school are considered. However, multi depots and multi schools have to be considered in real world. So, the modification of the proposed model is inevitable in order to reflect the reality of SBRP.

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REFERENCES


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