VOL. 2, NO. 11, October 2011 ISSN 2079-8407
Journal of Emerging Trends in Computing and Information Sciences

http://www.cisjournal.org

Inertia Optimization of a Two Masses System

[1]Mabrouki. H, [2]Firchichi. N

LR-ACEM- Department of electrical Engineering
Ecole Nationale d'Ingénieurs de Sfax, Route de soukra km 3 cite Elhabib 3052 Sfax Tunisia

[1]hichemmabrouki@gmail.fr , [2]bilelnoured@yahoo.fr

ABSTRACT

Generally, electric traction systems are complex because they consist of several electromechanical parameters that are in constant interaction. The multi-mass approach is used to identify these different parameters and the possibility of manipulation of these parameters in order to optimize the system and his operation.

Keywords: *Multi-mass approach, Traction systems, Two-mass system, Inertia Optimization*

1. INTRODUCTION

The use of electric drives especially in intermittent mode (the elevator, manipulators, storage, etc. ...), requires a high dynamic performance. These applications necessitate a greater interaction between the mechanical and electrical components [1]. Among the means to optimize energy, it emphasizes the optimization of start-up time and braking of induction motors resulting in variable costs and can reduce the loss of electrical power during the transient, reducing the temperature, wear, vibration, noise etc...

Several parameters have effects on the startup time and the dynamic behavior of systems; mention the constant electro-motor, the electromagnetic constant, and more [3]. The starting time still depends on the starting torque of the engine and the speed ratio of the speed-reduction gear, but also much of the inertia set games. It has been tested in this part to analyze and optimize the behavior of the system (mechanical model with two masses) by minimizing the torsion angles and from the moment of inertia of both masses [2]. To achieve this goal, we try to find a relationship that brings together read these four parameters. The method involves running the model with two masses to find a relationship between the torsion angles and inertia moments.

2. DEVELOPMENT OF THE MODEL WITH TWO MASSES STUDY OF TWO-MASS SYSTEM FOR OPTIMIZING

We have the following model of two-mass system (neglecting the damping coefficient) [6].

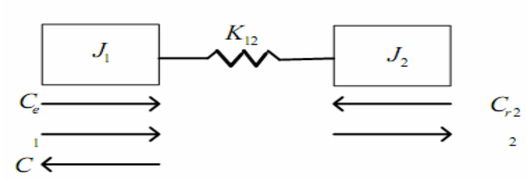

Fig.1: Equivalent circuit model with two masses unamortized.

The equation describing this system has two masses is [7]:

$$\begin{cases} J_1 \dfrac{d\Omega_1}{dt} + K_{12}R_1(R_1\theta_1 - R_2\theta_2) = C_{em} - C_{r1} \\ J_2 \dfrac{d\Omega_2}{dt} - K_{12}R_2(R_1\theta_1 - R_2\theta_2) = -C_{r2} \end{cases} \quad (1)$$

With:

J_1 : Moment of inertia of the first mass

J_2 : Moment of inertia of the second mass

Ω_1 : Angular velocity of the first mass

Ω_2 : Angular velocity of the second mass

K_{12} : Stiffness coefficient between the masses

R_1 : Wheel radius of the first mass

R_2 : Wheel radius of the second mass

VOL. 2, NO. 11, October 2011 ISSN 2079-8407
Journal of Emerging Trends in Computing and Information Sciences

http://www.cisjournal.org

θ_1 : Torsion angle of the first mass

θ_2 : Torsion angle of the second mass

And we have:

$$\frac{d\Omega_1}{dt} = \ddot{\theta}_1$$

$$\frac{d\Omega_2}{dt} = \ddot{\theta}_2$$

Either, if we assumed that;

$$X = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}, \ \dot{X} = \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix}, \ \ddot{X} = \begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix},$$

The model becomes:

$$\begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix}\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} K_{12} R_1^2 & -K_{12} R_1 R_2 \\ -K_{12} R_1 R_2 & K_{12} R_2^2 \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} C_{em} - C_{r1} \\ -C_{r2} \end{bmatrix} \quad (2)$$

Additionally, if we assumed that:

$$M = \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix},$$

$$K = \begin{bmatrix} K_{12}\ R_1^2 & -K_{12}\ R_1 R_2 \\ -K_{12}\ R_1 R_2 & K_{12}\ R_2^2 \end{bmatrix}$$

And, $F = \begin{bmatrix} C_{em} - C_r \\ -C_{r2} \end{bmatrix}$

The reduced form of the system becomes:

$$M\ddot{X} + KX = F \quad (3)$$

The equation (3) can be written as:

$$\ddot{X} + M^{-1}KX = M^{-1}F \quad (4)$$

If it is assumed that:

$X = PY$ Equivalent to: $\ddot{X} = P\ddot{Y}$

So the system becomes:

$$P\ddot{Y} + M^{-1}KPY = M^{-1}F \quad (5)$$

Whether:

$$\ddot{Y} + P^{-1}M^{-1}KPY = P^{-1}M^{-1}F \quad (6)$$

With P is the eigenvector.

The system of equations describing the dynamic behavior of the mechanical system is as follows:

$$\begin{cases} \ddot{Y}_1 + \omega_{01}^2 Y_1 = F_1 \\ \ddot{Y}_2 + \omega_{02}^2 Y_2 = F_2 \end{cases} \quad (7)$$

Therefore, $M = \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix}$

We have so: $M^{-1} = \begin{bmatrix} 1/J_1 & 0 \\ 0 & 1/J_2 \end{bmatrix}$

And; $K = \begin{bmatrix} K_{12}R_1^2 & -K_{12}R_1R_2 \\ -K_{12}R_1R_2 & K_{12}R_2^2 \end{bmatrix}$

Then:

$$M^{-1}K = \begin{bmatrix} \frac{K_{12}R_1^2}{J_1} & \frac{-K_{12}R_1R_2}{J_2} \\ \frac{-K_{12}R_1R_2}{J_1} & \frac{K_{12}R_2^2}{J_2} \end{bmatrix}$$

So the characteristic polynomial is:

$$\lambda^2 - \lambda\left(\frac{K_{12}R_1^2}{J_1} + \frac{K_{12}R_2^2}{J_2}\right) + \frac{K_{12}R_2^2R_1^2}{J_2} - \frac{K_{12}R_2^2R_1^2}{J_2} = 0$$

This implies that:

$$\lambda^2 - \lambda\left(\frac{K_{12}R_1^2}{J_1} + \frac{K_{12}R_2^2}{J_2}\right) = 0 \quad (8)$$

Solving this quadratic equation gives:

VOL. 2, NO. 11, October 2011 ISSN 2079-8407
Journal of Emerging Trends in Computing and Information Sciences

http://www.cisjournal.org

$\lambda = 0$ Where $\frac{K_{12}R_1^2}{J_1} + \frac{K_{12}R_2^2}{J_2} = 0$

If we find that one of the eigenvalues is zero, as in our case, we can conclude that the system is rigid, i.e. it is a movement of a single block or consists of a single mass purely rigid link between its elementary masses. Hence, with this condition, the study is not possible, since, firstly in this case we cannot determine a relationship that expresses the torsion angles based on the moments of inertia. Secondly this model hides the real dynamic behavior of the system [4].

3. CHANGING THE SYSTEM OF TWO MASSES FOR THE OPTIMIZATION

So, in order to solve this problem it is assumed that the system is embedded in a frame, thus adding another factor that binds the stiffness K frame and the second mass.

For the latter, and in order to study our system we made a small change by adding another link that binds the second mass with a fixed frame.

The following figure gives an overview of the system after the change.

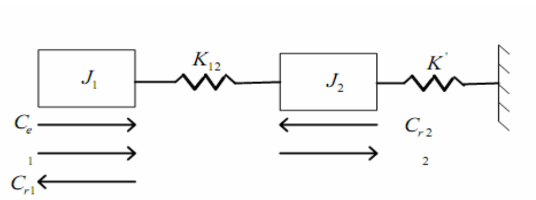

Fig.2: Equivalent circuit diagram with two masses with an embedded frame

$K^{'}$ is defined by the stiffness coefficient linking the second mass and the frame. The system of equations describing the operation of our system is:

$$\begin{cases} J_1 \dfrac{d\Omega_1}{dt} + K_{12}R_1\left(R_1\theta_1 - R_2\theta_2\right) = C_{em} - C_{r11} \\ J_2 \dfrac{d\Omega_2}{dt} - K_{12}R_2\left(R_1\theta_1 - R_2\theta_2\right) + K^{'}R_2^2\theta_2 = -C_{r2} \end{cases} \quad (9)$$

If we write our system in matrix form we obtain:

$$\begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix}\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} K_{12}R_1^2 & -K_{12}R_1R_2 \\ -K_{12}R_1R_2 & K_{12}R_2^2 + K^{'}R_2^2 \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} C_{em} - C_{r1} \\ -C_{r2} \end{bmatrix} \quad (10)$$

Following the same approach as before, we get:

$$M^{-1}K = \begin{bmatrix} \dfrac{K_{12}R_1^2}{J_1} & \dfrac{-K_{12}R_1R_2}{J_2} \\ \dfrac{-K_{12}R_1R_2}{J_1} & \dfrac{K_{12}R_2^2}{J_2} + \dfrac{K^{'}R_2^2}{J_2} \end{bmatrix} \quad (11)$$

The characteristic polynomial of this matrix is:

$$\lambda^2 - \lambda\left(\frac{K_{12}R_1^2}{J_1} + \frac{K_{12}R_2^2}{J_2} + \frac{K^{'}R_2^2}{J_2}\right) + \frac{K_{12}K^{'}R_2^2R_1^2}{J_2} = 0$$

Thus:

$$\Delta = \left(\frac{K_{12}R_1^2}{J_1} + \frac{K_{12}R_2^2}{J_2} + \frac{K^{'}R_2^2}{J_2}\right)^2 - 4\frac{K_{12}K^{'}R_2^2R_1^2}{J_2} = 0$$

Equal also:

$$\Delta = \left(\frac{K_{12}R_1^2}{J_1} + \frac{K_{12}R_2^2}{J_2} + \frac{K^{'}R_2^2}{J_2}\right)^2 + 4\frac{K_{12}K^{'}R_2^4}{J_2} = 0$$

Note that now, this expression is always positive, we can define roots of the characteristic polynomial as follows:

$$\lambda_1 = \frac{1}{2}\left(\frac{K_{12}R_1^2}{J_1} + \frac{K_{12}R_2^2}{J_2} + \frac{K^{'}R_2^2}{J_2}\right) - \sqrt{\Delta}$$

$$\lambda_2 = \frac{1}{2}\left(\frac{K_{12}R_1^2}{J_1} + \frac{K_{12}R_2^2}{J_2} + \frac{K^{'}R_2^2}{J_2}\right) + \sqrt{\Delta}$$

We see that λ_1 and λ_2 are positive, so the system studied is an oscillating system [5].

In order to determine the eigenvectors are defined by $\vec{V}_1$ and $\vec{V}_2$ which correspond to eigenvalues λ_1 and λ_2 we follow the following steps:

Either:

$$\left(M^{-1}k - \lambda_1 I\right)\vec{V}_1 = 0$$

VOL. 2, NO. 11, October 2011 ISSN 2079-8407
Journal of Emerging Trends in Computing and Information Sciences

With I is the identity matrix of dimension (2, 2) and $\vec{V}_1$ is the eigenvector. In fact we have:

$$\begin{bmatrix} \frac{K_{12}R_1^2}{J_1} & \frac{-K_{12}R_1R_2}{J_2} \\ \frac{-K_{12}R_1R_2}{J_1} & \frac{K_{12}R_2^2}{J_2}+\frac{K^{'}R_2^2}{J_2} \end{bmatrix} - \begin{bmatrix} \frac{1}{2}\left[\frac{K_{12}R_1^2}{J_1}+\frac{K_{12}R_2^2}{J_2}+\frac{K^{'}R_2^2}{J_2}-\sqrt{\Delta}\right] & 0 \\ 0 & \frac{1}{2}\left[\frac{K_{12}R_1^2}{J_1}+\frac{K_{12}R_2^2}{J_2}+\frac{K^{'}R_2^2}{J_2}-\sqrt{\Delta}\right] \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

What is involved:

$$\begin{bmatrix} \frac{1}{2}\left[\frac{K_{12}R_1^2}{J_1}-\frac{K_{12}R_2^2}{J_2}-\frac{K^{'}R_2^2}{J_2}+\sqrt{\Delta}\right] & -\frac{K_{12}R_2R_1}{J_2} \\ -\frac{K_{12}R_2R_1}{J_2} & \frac{1}{2}\left[-\frac{K_{12}R_1^2}{J_1}+\frac{K_{12}R_2^2}{J_2}+\frac{K^{'}R_2^2}{J_2}-\sqrt{\Delta}\right] \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Since the determinant of the matrix is zero, we can conclude that the equation that defines a_1 and b_1 is related, so it suffices to determine $a_1 = f(b_1)$ to determine the vector $\vec{V}_1$ we have:

$$\frac{1}{2}\left[\frac{K_{12}R_1^2}{J_1}-\frac{K_{12}R_2^2}{J_2}-\frac{K^{'}R_2^2}{J_2}+\sqrt{\Delta}\right]a_1 - \frac{K_{12}R_2R_1}{J_2}b_1 = 0$$

That is to say:

$$a_1 = \frac{-\frac{K_{12}R_2R_1}{J_2}b_1}{\frac{1}{2}\left[\frac{K_{12}R_1^2}{J_1}-\frac{K_{12}R_2^2}{J_2}-\frac{K^{'}R_2^2}{J_2}+\sqrt{\Delta}\right]}$$

So,

$$\vec{V}_1 = \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} \frac{-\frac{K_{12}R_2R_1}{J_2}b_1}{\frac{1}{2}\left[\frac{K_{12}R_1^2}{J_1}-\frac{K_{12}R_2^2}{J_2}-\frac{K^{'}R_2^2}{J_2}+\sqrt{\Delta}\right]} \\ b_1 \end{bmatrix}$$

We can take:

$$\vec{V}_1 = \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} \frac{-\frac{K_{12}R_2R_1}{J_2}}{\frac{1}{2}\left[\frac{K_{12}R_1^2}{J_1}-\frac{K_{12}R_2^2}{J_2}-\frac{K^{'}R_2^2}{J_2}+\sqrt{\Delta}\right]} \\ 1 \end{bmatrix}$$

We apply the same work it takes to determine $\vec{V}_2$:

$$\vec{V}_2 = \begin{bmatrix} 1 \\ \frac{-\frac{K_{12}R_2R_1}{J_2}}{\frac{1}{2}\left[\frac{K_{12}R_1^2}{J_1}+\frac{K_{12}R_2^2}{J_2}+\frac{K^{'}R_2^2}{J_2}-\sqrt{\Delta}\right]} \end{bmatrix}$$

VOL. 2, NO. 11, October 2011 ISSN 2079-8407
Journal of Emerging Trends in Computing and Information Sciences

http://www.cisjournal.org

So the matrix of eigenvectors P is equal to:

$$P=\begin{bmatrix} \dfrac{\dfrac{K_{12}R_2R_1}{J_2}}{\dfrac{1}{2}\left[\dfrac{K_{12}R_1^2}{J_1}-\dfrac{K_{12}R_2^2}{J_2}-\dfrac{K'R_2^2}{J_2}+\sqrt{\Delta}\right]} & 1 \\ 1 & \dfrac{\dfrac{K_{12}R_2R_1}{J_2}}{\dfrac{1}{2}\left[\dfrac{K_{12}R_1^2}{J_1}+\dfrac{K_{12}R_2^2}{J_2}+\dfrac{K'R_2^2}{J_2}-\sqrt{\Delta}\right]} \end{bmatrix}$$

Based on the calculation of diagonalization we find the following system:

$$\begin{cases} Y_1''+\omega_{01}^2\,Y_1=\dfrac{\left(\dfrac{K_{12}R_2R_1}{J_2}\right)\dfrac{1}{2}\left[\dfrac{K_{12}R_1^2}{J_1}-\dfrac{K_{12}R_2^2}{J_2}-\dfrac{K'R_2^2}{J_2}+\sqrt{\Delta}\right]C_{em}}{\dfrac{1}{4}\left[\dfrac{K_{12}R_1^2}{J_1}-\dfrac{K_{12}R_2^2}{J_2}-\dfrac{K'R_2^2}{J_2}+\sqrt{\Delta}\right]^2+\left(\dfrac{K_{12}R_2R_1}{J_2}\right)^2} \\ Y_2''+\omega_{02}^2\,Y_2=\dfrac{\dfrac{1}{4J_1}\left[\dfrac{K_{12}R_1^2}{J_1}-\dfrac{K_{12}R_2^2}{J_2}-\dfrac{K'R_2^2}{J_2}+\sqrt{\Delta}\right]^2C_{em}}{\dfrac{1}{4}\left[\dfrac{K_{12}R_1^2}{J_1}-\dfrac{K_{12}R_2^2}{J_2}-\dfrac{K'R_2^2}{J_2}+\sqrt{\Delta}\right]^2+\left(\dfrac{K_{12}R_2R_1}{J_2}\right)^2} \end{cases}$$

With: $\omega_{01}^2=\lambda_1$ and $\omega_{02}^2=\lambda_2$

The resolution of the previous system led to an expression that binds the torsion angles and moments of inertia, and from this relationship we can optimize operation of our system. Industrial experience shows that there are still cases of damage to occur during the startup phase. It turns out that the functioning and duration of life depend to a large extent on how the inertia is introduced. Through this, it was shown that by acting on the torsion angles, we can optimize the inertia set games.

3.1. Example of applying the model with two masses

We can consider a system of transmission of mechanical motion (multiplier or gear), as a mechanical model with two masses. This type of system is frequently used in industrial processes. Typical applications of this system include those chains conversion of wind energy. So for wind turbines that use the speed multipliers, we can use the study on two masses model to improve the conditions of operation of the wind [8]. To this end we end up with an energy-efficient mechanical system and in general, to win the chain complete.

According to the above, we can illustrate that the multi-mass approach application is very important in the sense of facilitating the understanding of a complex system like, for example, the wind power system.

.

4. CONCLUSION

If the system is rigid, i.e. it is a movement of a single block or one mass consisting of a link between purely rigid mass elements, the dynamic behavior of the real system is full of illusion.
So to remedy this problem, and to improve the functioning of an electric drive we take account for another model of the system, therefore, have been shown to optimize the operation of a drive, you must act on the stiffness of the transmission elements.

REFERENCES

[1] Ferchichi Noureddine, « Augmentation de l'efficacité des systèmes électromécaniques opérant en modes intermittents». Thèse de Doctorat, Académie Polytechnique de Minsk, 1998, p 273.

[2] Mabrouki .H (2007) « Optimisation énergétique d'un moto-réducteur, mécanisme d'application par l'approche mutimasses » Mémoire de Mastère en électrotechnique et systèmes indutriels (ESI) , Ecole Supérieure des Sciences Et Techniques de Tunis (ESSTT), Tunisia.

[3] DE Fornel B. (1994), Commande des machines électriques .Revue générale d'électricité No 8/94,33-35

[4] Jacques PLUSQUELLEC, Professeur à l'Institut Universitaire de Technologie de Cachan (Université Paris - Sud), «Vibrations», Techniques de l'Ingénieur, traité Sciences fondamentales.

[5] Catherine POTEL Philipe GATGNOL « Cours de mécanique de vibrations», Université de Maine - UFR Sciences et Technique.

[6] M.L Doumbia, G.Roy, V.Rajagopalan and V.Missurenko, «Comparative study of Multi-masse Models of electrical drives with asynchronous motors», Can.J.Elect. & comp. Eng, Vol.23 no.3, 1998.

[7] M.L Doumbia, Roy G, Rajagopalan V, (1997). « An Integrated solution for simulating Electrical drives systems with MATLAB/SIMULINK. IEEE International Symposium on Industrial electronics, 952-955.

[8] N.Laverdure ,S.Bacha, D.Roy, B.Raison and F.Dumas « Element of modeling of wind power systems with energy management : two structures in comparison» IEEE 28 th Annual Conference of the industrial Electronics Society , vol 2 05-08 November 2002 ,pp1083-1088.

BIOGRAPHIES

Mabrouki Hichem was born in Tunis, Tunisia, on May 22, 1979. He received the M.S. degree in Electrical Engineering from the Ecole Supérieure des Sciences et Techniques de Tunis (ESSTT), Tunisia in 2004. Also, he received the master degree in electrical engineering and industrial systems in 2007 from Ecole Supérieure des Sciences Et Techniques de Tunis (ESSTT), Tunisia. In September 2008, he joined the Department of Electrical Engineering at Institut Supérieur des Etudes technologiques de Kairouan, Tunisia, as an assistant professor. He is now a researcher in Laboratory of Advanced Control and Energy Management) (ACEM) in ENIS. He works in the Ph.D. theses degree in modeling of electrical cables for the study of electrical faults on a network of decentralized production.

Firchichi. Noureddine : bilelnoured@yahoo.fr